AN ANALYSIS OF SOME CURRENT AND PROSPECTIVE CONTROL SYSTEMS FOR A NEAR-SURFACE SUBMARINE

John Daniels Blosser

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by

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Table of Contents

Nomenclature

x,y,z	rectalinear spacial coordinates
z_o	depth of body
u,v,w	velocity components in x,y,z, directions
u_o,v_o,w_o	water particle orbital velocities
p,q,r	roll, pitch, yaw angular velocities
g	universal gravity constant
f	linear frequency Hz
ω	angular frequency
$\Delta\omega$	bandwidth
ω_e	frequency of encounter
Δf	frequency shift
T	wave period and time to encounter
A	surface wave amplitude
h	surface wave height (crest to trough)
λ	wave length
k	wave number
V_p	wave phase velocity
V_s	ship velocity
ϵ	arbitrary phase angle
ϕ	velocity potential and angle in horizonal plane
p	pressure
p_{atm}	atmospheric pressure 0psig
ρ	density
E	energy
E_{tot}	total Energy
μ	incident angle between body and waves (180° in head seas)
$\vec{F}$	total force vector
Z	heave force
M	pitch moment
$\forall$	volumetric displacement
CG	center of gravity
CB	center of buoyancy
CP	center of pressure
α	angle of attack
δ	control surface deflection angle
S	stiffness coefficient
θ	pitch angle or other angle in vertical plane
C_L	lift coefficient
C_o	C_L intercept
C_α	C_L slope
L	lift
c	speed of sound
b,c,d,m,n,s	lengths
$\dot{}$	derivative with respect to time
$\bar{}$	average taken over time

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Submitted to the Department of Ocean Engineering on May 11, 1973, in partial fulfillment of the requirements for the degree of Master of Science in Naval Architecture and Marine Engineering.

Thesis Supervisor: Martin A. Abkowitz
Title: Professor of Ocean Engineering

Abstract

Based upon studies of phenomena affecting near-surface submarine control, the author has analyzed the existing method of control as well as three hypothetical systems. Though the analysis in some cases was necessarily qualitative, the final system is considered to be a plausible solution to the control problem. It employs the advantageous qualities of the first three in a more efficient combination.

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Introduction

Operational experience has shown that there is a definite need for a new or at least improved method for control of a near-surface submarine. This need has been documented in the form of an unacceptable number of broachings by certain classes of the Navy's nuclear submarines. Additionally, it is hoped that an improved sea state capability and greater accuracy can be attained.

"Near-surface" generally refers to a submarine which is close enough to the surface to be affected by surface waves. For the purposes of this paper, a submarine will be considered near the surface if it is less than one half of a wave length from the surface. The validity of this statement will be verified later.

The subject of optimum near-surface control of a submarine is a relatively new one. As yet, many of the phenomena governing a submarine's response to submerged turbulence is not completely understood. Unfortunately, much of the experimental data which has been collected in attempts to gain a better understanding of this subject has been classified by the government.

The purpose of this paper, therefore, is not to design the optimum near-surface control system. Rather it is an attempt to study and comprehend the nature of the problem and some of the possible (not necessarily optimal) solutions. It has been somewhat restricted to a qualitative approach with the use of examples where they are helpful and possible.

Initally the reasons for a submarine to even be near the surface are presented in the hope of gaining some insight into the control requirements which will to a large extent determine what type of control systems are acceptable and what degree of accuracy is needed.

This is followed by a section defining and describing some of the applicable qualities of surface excitations - waves. The next section then shows in an orderly manner the effects these waves have on submerged body. The analysis is begun by examining the terms of Bernoulli's equation for irrotational flow in an inviscid fluid.

$$p = -\rho\left[\frac{\partial \phi}{\partial t} + \frac{1}{2}V^2 + gz\right] \tag{1}$$

It is the integral of this pressure over the surface of the body which causes the forces and moments responsible for body motions.

Before investigating the four different control systems presented, a brief discussion of control systems in general is included. This emphasizes the two primary types of systems, open and closed loop, and defines several of the basic parameters which are indicators of the quality of the system.

Finally, the present system is analyzed in light of the preceding sections. Three alternative systems are then presented. The first one merely replaces the man in the control loop with a computer in an attempt to reduce lag time. The next system essentially begins from scratch, using a means of detecting the approaching waves so that computers can be used

to predict the motion of the boat before it occurs and take the necessary action to counter. The last system is more or less a compromise of the first three in the hope that the good features of each can be combined to provide a system better than any of its predecessors.

Conclusions and recommendations by the author conclude the main body of the paper. However, three appendices are included which describe a possible sonar system to beused in wave prediction, the classical approach of using the linearized equations of motion, and lastly linear strip theory.

Missions of a Near-surface Submarine

Before attempting to design any type of control system, it is imperative that the mission requirements be examined. These requirements inevitably place certain restrictions on various control parameters.

Considering only the case of the near-surface submarine in straight ahead motion, the following situations are noteworthy because all military submarines must be capable of at least one of these and possibly all of them.

1. Periscope depth for navigational star sightings or observation of navigational satellites: This is one of the least demanding situations. Here the most convenient heading and speed may generally be chosen without regard to cavitation or fear of audio detection. (If such detection is a threat, a submarine is not likely to chance periscope depth merely for a look at the stars.) Similarly, it would probably not be necessary to come up in extreme sea states.

2. Pre-torpedo run optical sightings: In most present day cases it may not be necessary to make this run. Consequently, in severe seas it would be possible to fire the torpedoes without coming near the surface. However, an optical sighting can certainly increase the probability of a hit as well as positively identify the adversary. For this reason a high sea state capability is advisable. In order to remain undetected, speed must remain low enough so as to avoid cavitation. Sonar transmissions must either be stopped or carefully guarded for

similar reasons. Low speeds mean less dynamic control from the planes thus compounding the rough sea problem. Finally, it would be ridiculous to require skippers to assume a particular course with respect to the direction of the prevailing seas when engaged in combat.

3. Special operations: Considering only deep water special operations such as reconnaissance, it would again be necessary to impose requirements similar to those listed for a torpedo run.

4. Transmission: Again in the case of reporting an enemy's position, it would become necessary to use the torpedo run restrictions. Additionally, it is desirable to be able to limit pitch and heave amplitudes to a specified amount in order to keep antennae above water thereby insuring continuous transmission.

5. Missle launchings: Missle launching is currently a very special case since it is normally done from a hovering position. In such a case, the control planes are ineffective. This particular case will not be considered. However, if missles are eventually to be launched with way on, a control system must be capable of control at low speeds and in high seas as well as for any direction relative to the prevailing seas.

Characteristics of the Excitation

When considering the control of any vessel, it is necessary to analyze not only the various dynamic properties of the vessel but also those of the environment in which the vessel is to operate.

In the specific case of the near-surface submarine, the environment is of particular importance. Unlike the deeply submerged submarine, which can ignore the relative insignificant or even nonexistent effects of surface waves, the near-surface submarine is faced with a very serious control problem. The submerged wave problem is compounded in comparison with that encountered by the surface ship since the operator can not visually detect the approaching disturbance.

There are basically two approaches for analyzing a seaway. Because the properties of a sinusoidal plane progressive wave are well defined and easily modeled in the towing tank, such a representation has gained wide usage. Additionally, linear response theories (the ratio of ship heave, etc., to wave amplitude for a given frequency is assumed constant) have proven to be quite accurate for most applications. The parameters and properties described in figure 1 are applicable for a sinusoidal wave on the free surface.

Use of the equations of motion for an ideal fluid and the appropriate boundary conditions are used to evaluate the velocity potential.

$$\phi = \frac{gA}{\omega} \frac{\cosh[K(z+d)]}{\cosh[Kd]} \cos(-\omega t + kx + \epsilon) \qquad (2)$$

where the phase difference, ϵ, is arbitrary. The terms involving the hyperbolic cosines cancel in deep water due to increasing depth, d. The wave amplitude is given by:

$$A = -A_{max} \sin(kx - \omega t + \epsilon) \tag{3}$$

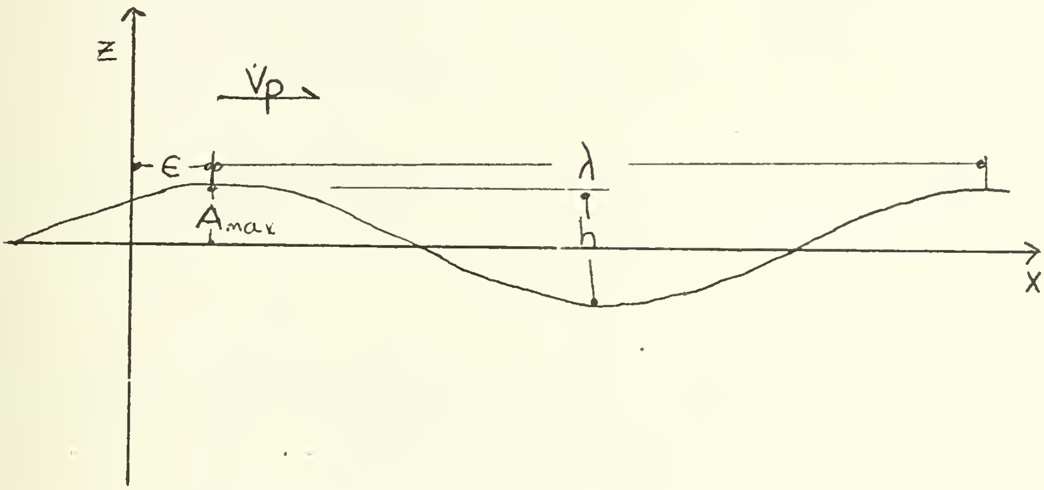


Figure 1

The following identities are also useful:

wave frequency, $\omega = \sqrt{2\pi g/\lambda}$ (4)

wave number, $k = \omega^2/g$ (5)

phase velocity (velocity of wave crests), $V_p = \omega/k$ (6)

wave period, $T = 2\pi/\omega$ (7)

The water particle orbital velocity at the surface can easily be seen to be $u = \omega A_{max} \sin(kx - \omega t + \epsilon)$ (8)

Properties below the free surface behave in an exponentially decaying manner to the corresponding properties on the free surface. (See figure 2)

$$u(x,z,t) = \omega A_{max} \exp[-2\pi z_o/\lambda] \sin(kx - \omega t + \epsilon) \tag{9}$$

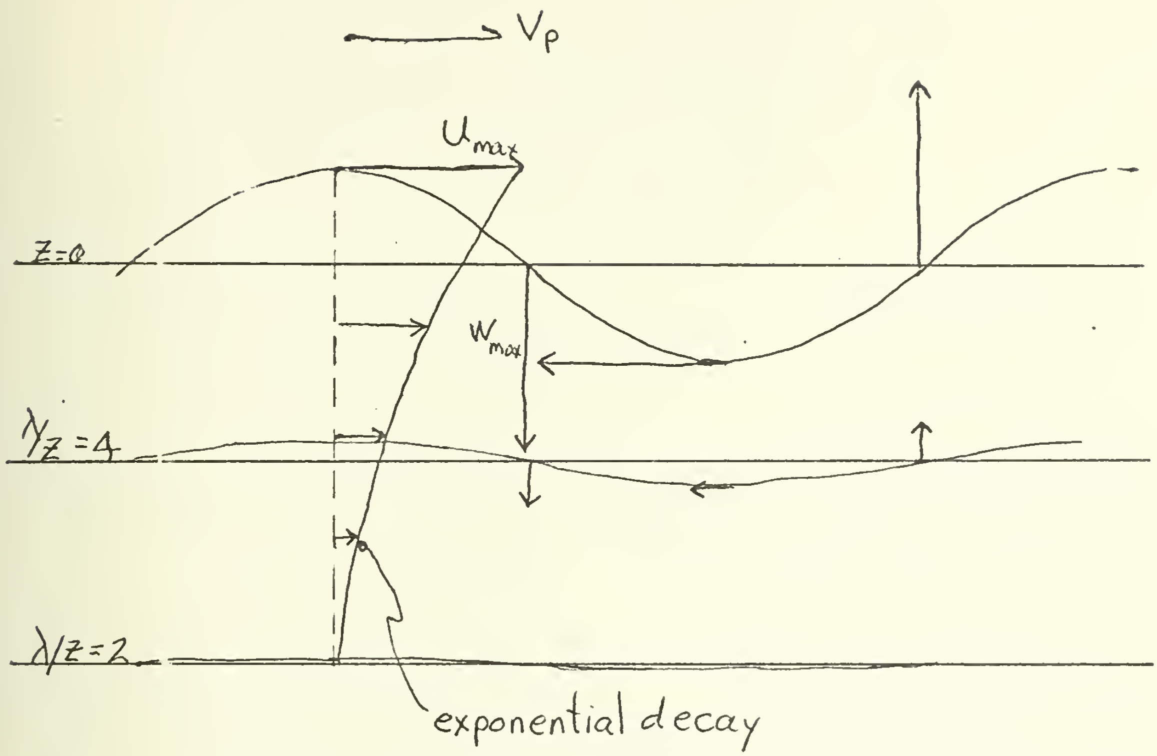


Figure 2

Such a model is indeed a powerful tool and will be used later. However, in order to gain additional information, it is, in the case of the near-surface submarine, helpful to undertake a more general approach. It is therefore necessary to model, simulate, or otherwise define a stochastic seaway.

In its simplest and most common form, this representation assumes the form of a series of superimposed sinusoidal waves each characterized by a frequency and amplitude. A very large number of these waves will in fact generate an irregular non-repeating pattern when they are added to each other in a linear manner. (See figure 3.) This randomness is further assured

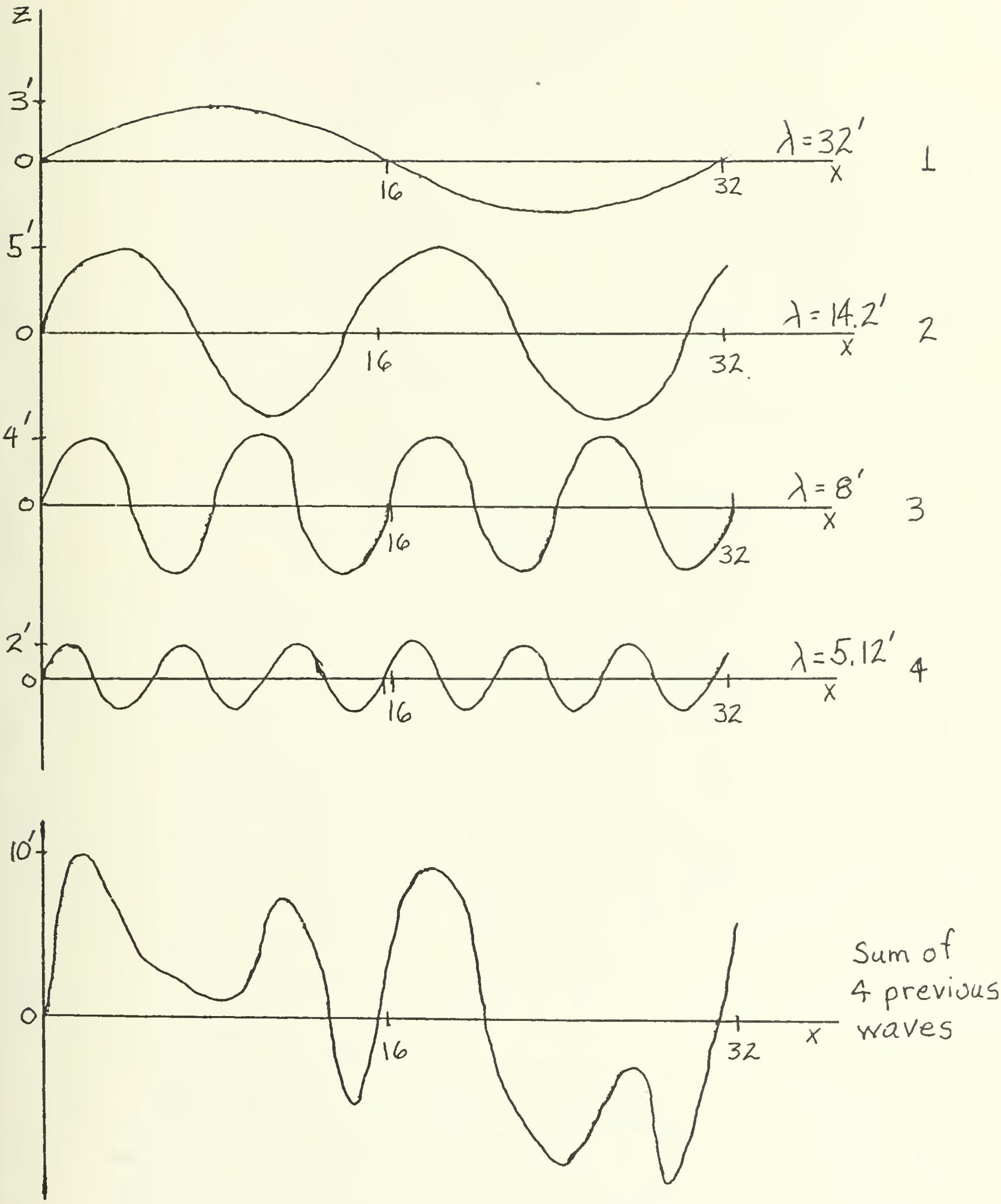

Z
3′
0
λ=32′
16
32
X
1
5′
0
λ=14.2′
16
32
X
2
4′
0
λ=8′
16
32
X
3
2′
0
λ=5.12′
16
32
X
4
10′
0
16
32
X
Sum of
4 previous
waves

Figure 3

since the phase relations between waves of different frequencies and heights in the real ocean are purely arbitrary. For this reason, it is impossible to predict whether interference among different waves will be constructive or destructive.

When an unknown number of waves with unknown amplitudes and frequencies combine, the result is not only an irregular seaway, but also an unpredictable one. There is however one way to comprehend and analyze this type of disturbance. That is to determine the total energy of the seaway.

An energy analysis of a a single sinusoidal wave results in the discovery that the energy is proportional to gravity, the density of the fluid, and the amplitude squared of the wave. It is independent of the wave length, period, or velocity. Specifically, the energy of a simple harmonic wave is $1/8\rho g h^2$ per square foot of sea surface. (Note: Wave height will be used instead of amplitude because it is more convenient for oceanographers to measure.) The energy of the entire seaway is merely the summation of the energies of each of the component waves linearly superimposed. The total energy may then be written as follows,

$$E_{TOTAL} = \frac{\rho g}{8} \sum_{i=1}^{\infty} (h_{\omega_i})^2 \qquad (10)$$

This distribution of energy according to discrete frequencies is known as the energy density spectrum of the sea. This spectrum may be plotted as energy versus wave frequency. For the four waves in figure 3, the spectrum can be computed and plotted in the following manner. (See also figure 4.)

wave number	λ(ft)	h(ft)	$\omega = \sqrt{2\pi g/\lambda}$(sec^{-1})	E(ft-lb-sec/ft^2)
1	32	3	2.51	72
2	14.2	5	3.77	200
3	8	4	5.03	128
4	5.12	2	6.28	32

$$E_{TOT} = \frac{\rho g}{8} \sum_{i=1}^{4} (h_{\omega_i})^2$$

$$E_{TOT} = \rho g/8\,[9 + 25 + 16 + 4]$$

$$E_{TOT} = 72 + 200 + 128 + 32 = 432 \text{ ft-lb-}\cancel{\text{sec}}/\text{ft}^2$$

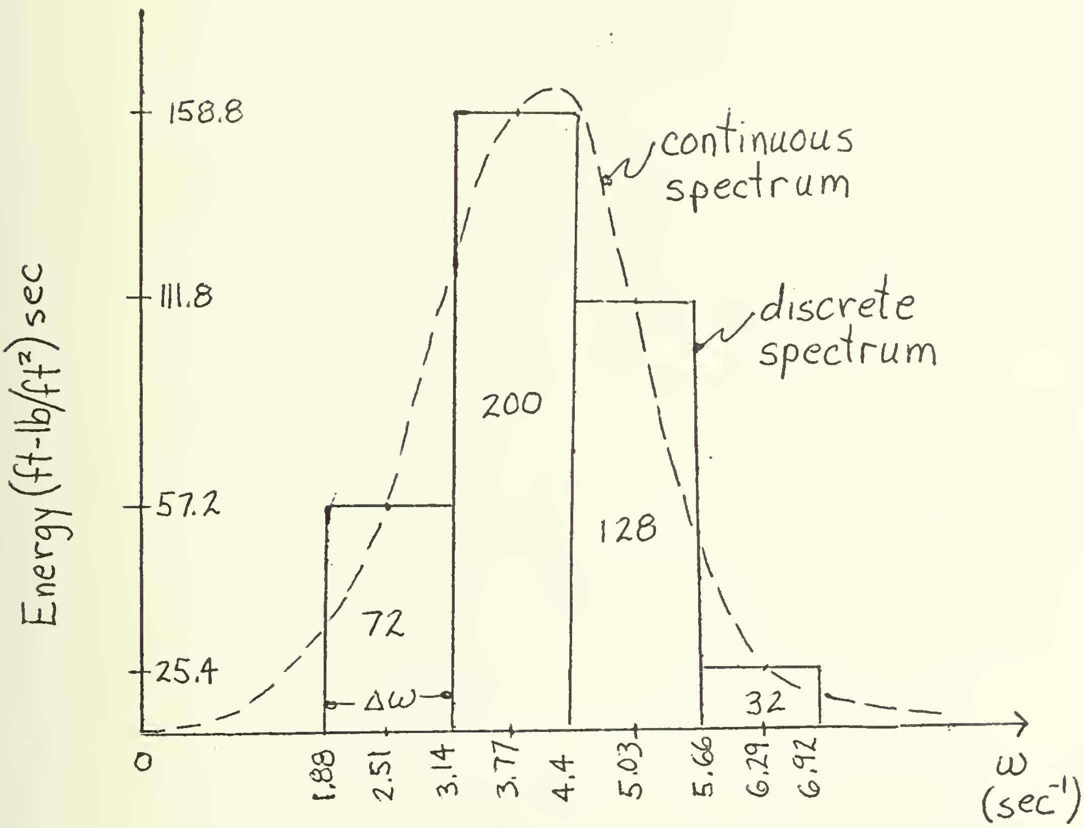


Figure 4

The total area within the four rectangles represents the total energy per square foot of the wave system. The width at the base of each rectangle, $\Delta\omega$, is known as the bandwidth. Since the real spectrum is composed of an infinite number of frequencies and phases, as it indeed must be if it is never to

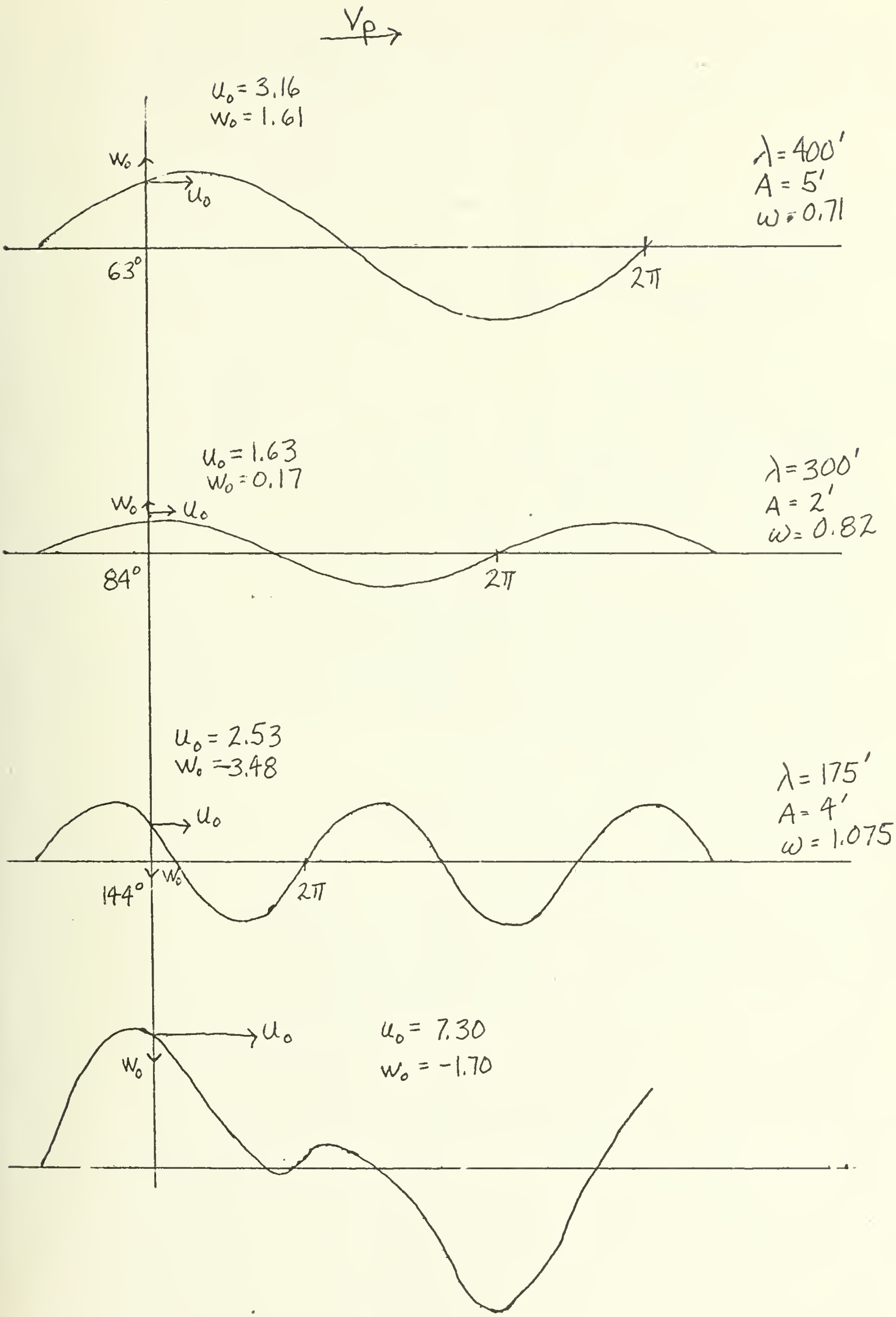
be repeated in time or space, $\Delta\omega$ must be made to approach zero. When this is done, a continuous curve is the result. This curve will change for different sea states in different parts of the world. However, in general as the winds and fetch increase, the peak of the spectrum will grow higher and tend to shift to the left and lower frequencies.[3]

For the case of a submarine moving ahead at a constant velocity, it is necessary to make one correction to this energy spectrum. This involves rescaling the frequency coordinate so that the frequency becomes the frequency relative to the moving boat. For head seas the frequency is increased and the shape of the curve tends to be stretched along the frequency axis. In general, the frequency of encounter is

$$\omega_e = \left[1 - \left(\omega V_{SHIP}/g\right) \cos\mu\right] \tag{11}$$

Lastly continuing with the assumption that the waves are of small amplitude and that they may be linearly superimposed, it is possible to predict the particle velocity at a given depth if the surface wave height above mean water level is known. This can be done by merely adding the velocity vectors of each of the component waves. To correct the resulting vector for depth, the same decaying factor is used, $\exp[-2\pi Z_o/\lambda]$.

It can be seen in figure 5 that if the orbital velocities of the components are known, then the orbital velocities of the resultant wave may be readily calculated. The problem then becomes one of obtaining orbital velocities if the component waves are unknown as in the real life case. Since it is now



V_p
$u_0 = 3.16$
$w_0 = 1.61$
w_0
u_0
$\lambda = 400'$
$A = 5'$
$\omega = 0.71$
63°
2π
$u_0 = 1.63$
$w_0 = 0.17$
w_0
u_0
$\lambda = 300'$
$A = 2'$
$\omega = 0.82$
84°
2π
$u_0 = 2.53$
$w_0 = -3.48$
u_0
$\lambda = 175'$
$A = 4'$
$\omega = 1.075$
144°
w_0
2π
u_0
$u_0 = 7.30$
$w_0 = -1.70$
w_0

Figure 5

known that linear superposition is a valid means of obtaining these velocities, another method of linear superposition may be used.

The kinematic boundary condition for a regular wave states that the normal velocity of the surfacr must be the same as the fluid velocity at that point. For waves with small slopes, this says that

$$\frac{\partial h}{\partial t} = \omega = \frac{\partial \phi}{\partial z} \qquad (12)$$

This must also be true for a series of regular waves. Continuous monitoring with a hydrostatic depth sensor will allow calculation of ω.

It is this vertical component of the orbital velocities which causes pitch and heave of the entire body and which affects the angle of attack on the control planes.

Environment Effects on the Near-surface Submarine

There are several factors which affect the control of a near-surface submarine. Any one of them taken separately could either be calculated and modeled or ignored. It is the aggregate sum of these factors which tends to complicate the design problem. The forces on a submerged body may be expressed as the integral of the pressure normal to the body taken over the surface of the body. Considering only the case of an ideal fluid with irrotation flow, the integration of Euler's equations yields the following expression for the pressure.

$$p = -\rho\left[\frac{\partial\phi}{\partial t} + \tfrac{1}{2}V^2 + gz\right] \qquad (13)$$

Looking first of all at the last term in the pressure equation, it is necessary to consider the constant pressure surfaces mentioned in the preceding section. The shape of these surfaces is dictated by the exponentially degenerating amplitude. For a submarine at a mean depth of 50 feet under a wave 628 feet long,

$$h(z) = h_{surface}\, \exp[-2\pi z_0/\lambda] \qquad (14)$$

$$h(50) = h_{surface}\, \exp[-2\pi \times 50/628]$$

$$h(50) = h_{surface}\, \exp[-0.5]$$

$$h(50) = h_{surface} \times 0.606$$

This is still a very significant wave amplitude, especially in moderate or high sea states. If the wave profile is frozen for an instant in time, the problem becomes one of hydrostatics in which

$$\vec{F} = \iint_S p\,\vec{n}\, dS \qquad (15)$$

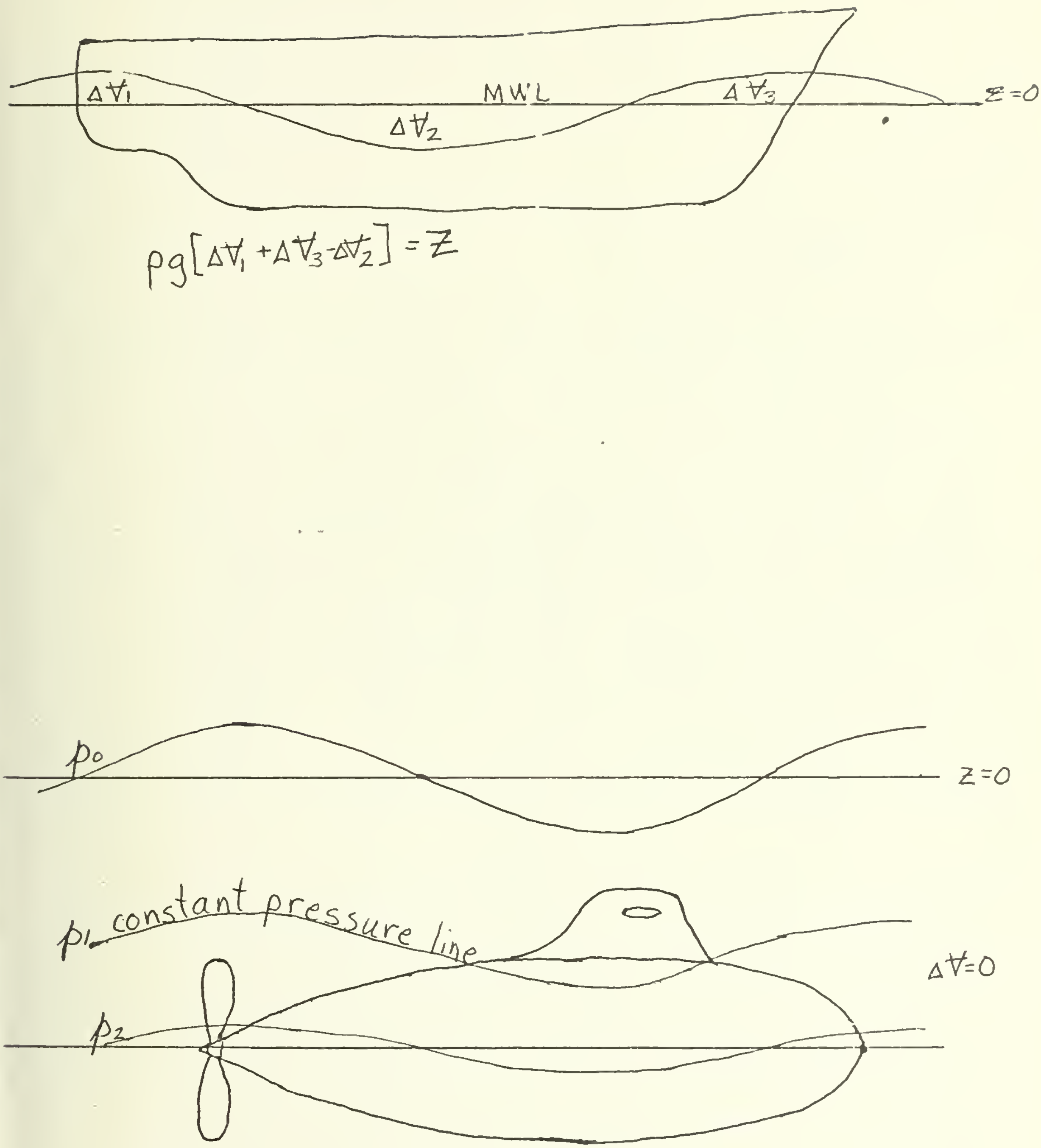

$\Delta\forall_1$
MWL
$\Delta\forall_3$
$Z=0$
$\Delta\forall_2$
$\rho g[\Delta\forall_1 + \Delta\forall_3 - \Delta\forall_2] = Z$
p_0
$Z=0$
p_1 constant pressure line
$\Delta\forall=0$
p_2

Figure 6

$$\vec{F}_{hydrostatic} = \iint_S -\rho g z \vec{n}\, dS$$
$$\vec{F}_{hs} = -\rho g \iint_S z(+\vec{n})\, dS$$
$$\vec{F}_{hs} = -\rho g \iiint_{\forall} \nabla \cdot \vec{z}\, dV$$
$$\vec{F}_{hs} = -\rho g \vec{k} \iiint_V dV$$
$$\vec{F}_{hs} = -\rho g \forall \vec{k} \qquad (16)$$

where $\vec{n}$ is the unit normal vector pointing into the body and $\vec{k}$ is the unit vector in the +z direction.[10]

Ideally, this passage of a pure pressure wave without the accompanying particle velocities would create essentially no change in force since the integral of the pressure would not change but remain constant. This phenomena is perhaps better understood by looking at a drawing of a surface and submerged vessel. In the first case, the vessel must heave and/or pitch to keep the submerged volume constant. This assures a balance in the force equation. In the second, surrounded by water, the volume does not change with the passage of a wave. (See figure 6.)

One of the most prominent effects is referred to by submariners as "suck" or the Venturi effect. There is indeed a component of the pressure equation which results in the submarine being sucked towards the surface. However, the reference to the Venturi effect is not a technically precise description. Strictly speaking, the Venturi effect is applicable only for rigid boundaries and steady flow. It in effect says that for the case where there is a constriction of the cross-sectional area of flow, there is a corresponding increase in

the velocity of the flow. This velocity increase results in a pressure decrease.

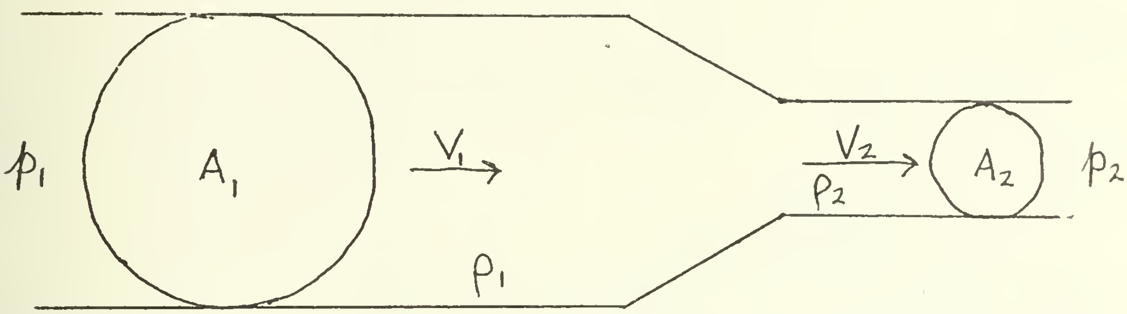


Figure 7

For steady flow, $p + \frac{1}{2}\rho V^2 = \text{constant}.$ (17)

Therefore, $p_1 + \frac{1}{2}\rho_1 V_1^2 = p_2 + \frac{1}{2}\rho_2 V_2^2.$ (18)

But conservation mass requires that $V_1 A_1 \rho_1 dt = V_2 A_2 \rho_2 dt.$

$\rho_1 = \rho_2$ for an incompressible fluid.

$$V_1 = (A_2/A_1) V_2 \qquad (19)$$

$$p_1 = p_2 + \frac{1}{2}\rho [V_2^2 - (V_2 A_2/A_1)^2]$$

$$p_1 = p_2 + \frac{1}{2}\rho V_2^2 [1 - (A_2/A_1)^2] \qquad (20)$$

This is only partly applicable for the near-surface submarine. Since the depth beneath a submarine may be assumed infinite, there will be no constriction of flow around the lower half of the submarine. Assuming that the submarine is in a uniform flow field with no disturbance, there will be a slight reduction in the cross-sectional area of flow around the upper half resulting in a higher velocity and thus a lower pressure. However, the reduction in cross-sectional area is not as great

as it might be due to the non-rigidity of the boundary. In fact, when the submarine passes beneath the undisturbed surface, it will create a wave on the surface which it pushes along with it. This wave tends to increase the cross-sectional area of flow such that the Venturi effect is considerably reduced. (See figure 8)

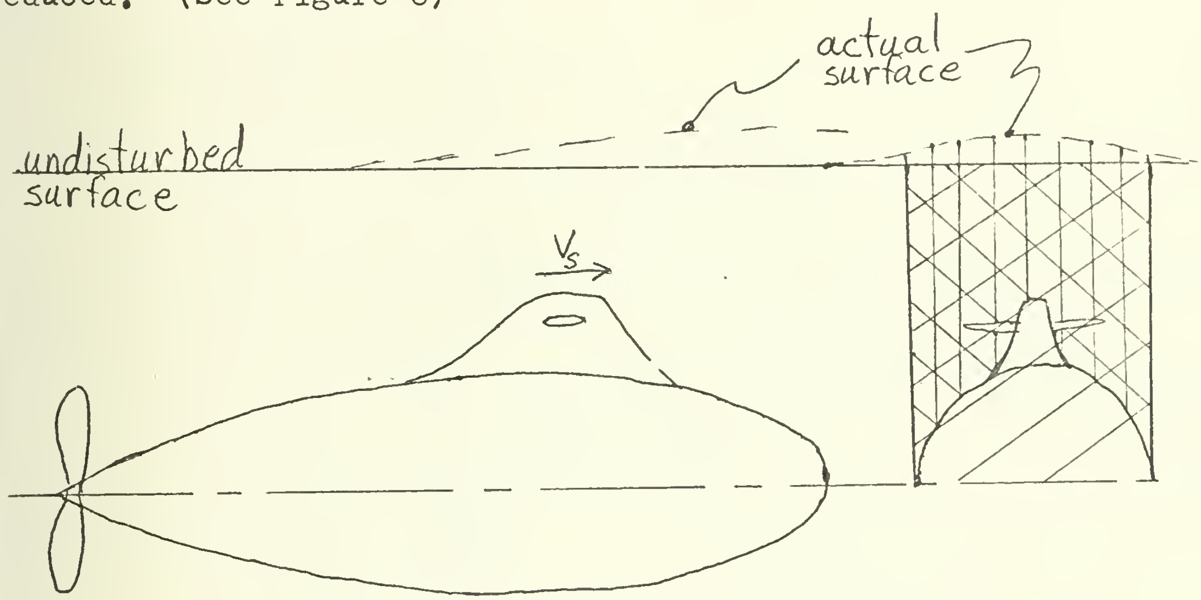


Figure 8

A_1 is the initial undisturbed cross-sectional area before the submarine enters the region.

A_2 is the new area which would exist if the surface were a rigid boundary.

A_3 is the actual area as a result of the non-rigidity of the surface. $A_1 > A_3 > A_2$

The second discrepancy in calling the "suck" a Venturi effect is the assumption of steady flow. In reality the

surface will normally have some random configuration which will result in the water particles having orbital velocities.

If the irregular seaway is broken into its regular sinusoidal components as before, it can be seen that the velocities of the orbital particles increase the magnitude of the V^2 term in the pressure equation.

$$V_{relative} = V_{ship} + u_o \cos(\omega t + \epsilon) \qquad (21)$$

where $u_o = \exp[-2\pi z_o/\lambda]\, u_{surface(max)}$ (22)

$$V_{rel.} = V_s + u_{max} \exp[-2\pi z_o/\lambda] \cos(\omega t+\epsilon)$$

$$V_{rel}^2 = V_s^2 + 2V_s u_{max} \exp[-2\pi z_o/\lambda] \cos(\omega t+\epsilon) + u_{max}^2 \exp[-4\pi z_o/\lambda] \cos^2(\omega t+\epsilon)$$

If V_{rel}^2 is averaged over time, the second term involving $\cos(\omega t+\epsilon)$ becomes zero. The final term becomes $\frac{1}{2} u_{max}^2 \exp[-4\pi z_o/\lambda]$.

So $V_{rel}^2 = V_s^2 + \frac{1}{2} u_{max}^2 \exp[-4\pi z_o/\lambda]$ (23)

For example, if $V_s = 6$ ft/sec

$z_o = 40$ ft

$\lambda = 628$ ft

$h = 12$ ft

$$V_{rel}^2 = 36 + \tfrac{1}{2}(\omega h)^2 \exp[-4\pi\, 40/628]$$

$$V_{rel}^2 = 36 + \tfrac{1}{2}\, 36 \times 2\pi g/628 \exp[-0.8]$$

$$V_{rel}^2 = 36 + 1.75$$

$$V_{rel}^2 = 36[1 + .05]$$

Initially, this may not seem like much, but in fact, this effect coupled with the previously mentioned Venturi effect can create as much as ten to twenty tons of lift on a submarine. This force is dependent upon the amplitude squared, upon the length of the waves in the sea state, and upon the mean depth of

the body. This force is defined as being a time averaged force which remains relatively constant through time provided that the sea state remains unchanged.

Since the suck force is dependent upon the sea state, the stochastic case can be modeled using an energy density analysis. The spectrum can provide a mean value for the frequency and the wave height. The mean or expected value of the frequency can be found much like the expected value of a probability density function.

$$\omega_{EXP} = \sum_{i=0}^{\ell} \omega_i [S(\omega_i)] \Big/ \sum_{i=0}^{\ell} S(\omega_i) \qquad (24)$$

The denominator is used to normalize the value.

$$\lambda_{EXP} = 2\pi g / (\omega_{EXP})^2 \qquad (25)$$

$$E_{TOT} = \tfrac{1}{8} \rho g (h_{EXP})^2 \qquad (26)$$

$$\text{so} \quad h_{EXP} = \sqrt{8 E_{TOT} / \rho g} \qquad (27)$$

The expected height and frequency can be combined to form an average or expected wave. This wave can be used to predict the average suck due to particle velocities as shown below.

$$\overline{P}_{orbital\ vel} = -\tfrac{1}{2} \rho \left[\tfrac{1}{8} \exp[-4\pi Z_0/\lambda] \, h_{EXP}^2 \, \omega_{EXP}^2 \right] \qquad (28)$$

and substitution into (15) gives

$$\overline{F}_{SUCK(o.v.)} = -\rho/16 \iint_S \exp[-4\pi Z_0/\lambda] \, h_{EXP}^2 \, \omega_{EXP}^2 \, dS \qquad (29)$$

Using strip theory (appendix 3)

$$\overline{F}_{SUCK(o.v.)} = -\rho/16 \sum_{i=1}^{n} \tfrac{1}{2} h_{EXP}^2 \, \omega_{EXP}^2 \int_0^{\Delta\ell}\int_0^{\pi} \exp[-4\pi(d - R\cos\theta)/\lambda] \, d\theta \, d\ell \qquad (30)$$

$$\overline{F}_{SUCK(o.v.)} = -\rho/8 \, h_{EXP}^2 \, \omega_{EXP}^2 \exp[-4\pi d/\lambda] \sum_{i=1}^{n} \int_0^{2\pi} \exp[-4\pi R\cos\theta/\lambda] \, d\theta \qquad (31)$$

where n is the number of sections each of which is a unit long.

The primary problem involved with the suck created by the free surface is that the term is second order in V and thus will not be accounted for in the normal first order equations of motion. Even if the equations are altered to account for second order terms, an additional problem arises. In all of the preceding analysis, it was assumed that the insertion of the submarine into the orbits of the particles did not affect these orbits. This is certainly a gross over-simplification of a very complex subject though necessary if useful results are to be obtained.

Upon encountering this force, a normally stable and neutrally buoyant body becomes unstable and seeks a new equilibrium position by ascending to the surface unless proper precautions are taken to prevent such mishaps. These precautions themselves create additional problems.

There are basically two ways to offset the suck of the surface. The most common is to take on ballast thereby making the boat negatively buoyant. The problem here is determining the proper amount of ballast for various sea states. Too much ballast can make the boat dangerously heavy and sluggish while too little may result in broaching.

The second method is to use an angle on the sail planes to create a downward force. The problem here being a reduction in control authority and the possibility of losing the downward force if the orbital particle velocity is such that the angle of attack on the planes is reduced to zero.

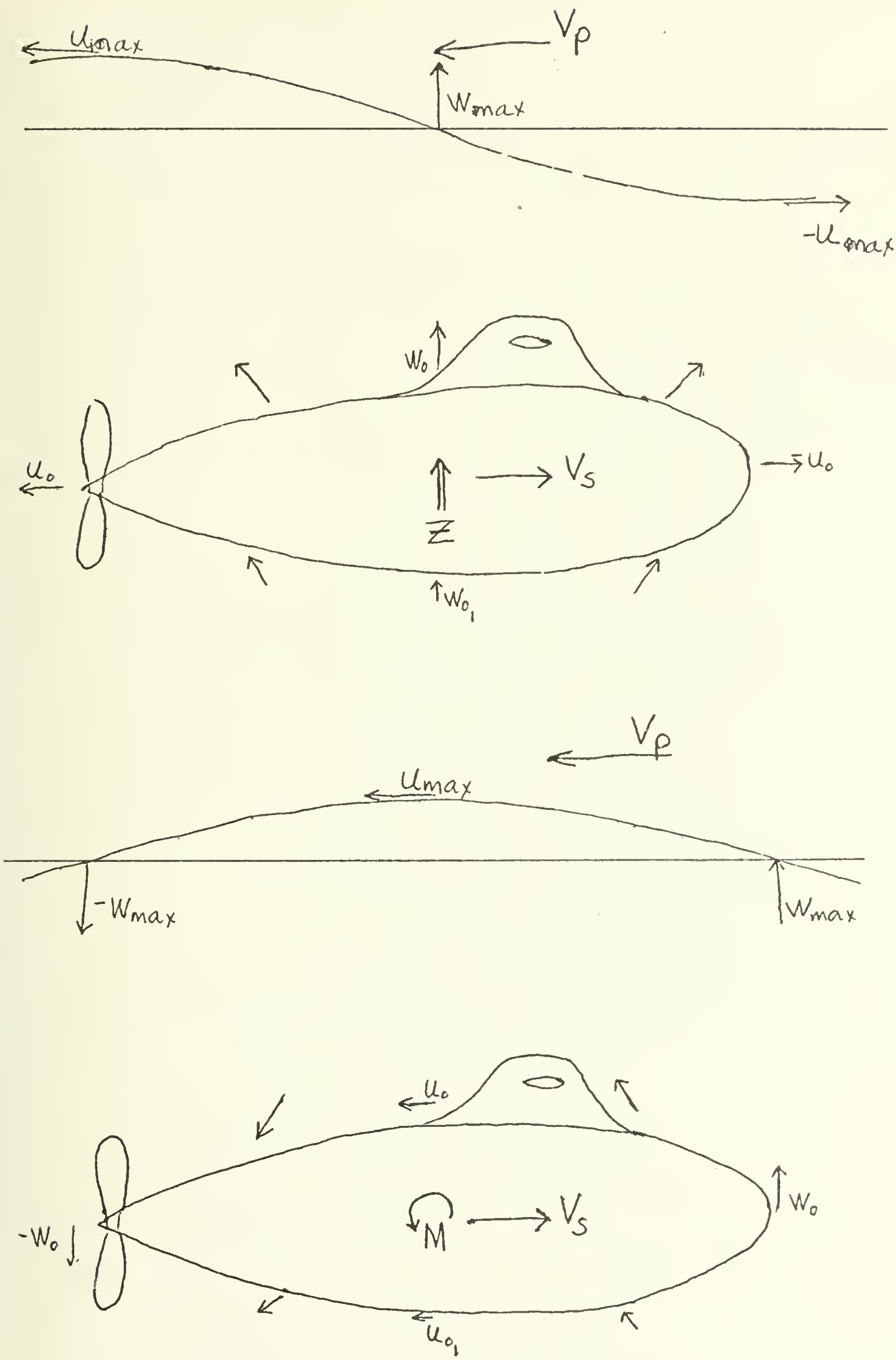

Umax
Vp
Wmax
-Umax
Wo
uo
Vs
Z
uo
Wo1
Vp
Umax
-Wmax
Wmax
uo
Wo
-Wo
M
Vs
uo1

Figure 10

In addition to the time averaged suck experienced by the body due to the orbital velocity, the particles also induce time dependent forces and moments, pitch and heave in particular. The exact effects of the orbital velocity components on the hull are very difficult to analyze precisely due to the many approximations and assumptions which must be made to keep the theory tractable. However, a couple of simple examples will serve to show at least qualitatively what happens as a submarine passes under a regular sinusoidal wave. (See figure 10)

As stated in the section on waves, the direction of the vectors representing the particle velocity vary sinusoidally with time and the coordinate x, and they vary in magnitude exponentially with depth. From figure 10 a., it is observed that when the body passes under the trailing half of a wave crest it is subjected to velocity vectors which add up to yield a net force in an upward direction. Similarly when the body passes beneath the leading half of the wave, it experiences a net downward force. From figure 10 b., it is seen that if the body is directly under a crest or a trough, one half of the body experiences an upward force and the other half experiences a downward force. These two forces form a couple to create a pitching moment. In the two pictures, the wave length is approximately twice the length of the boat. Intuitively, this would seem to create the worst heave and pitch. In actuality this is not far wrong. Experimental model tests run for the U. S. Navy's Bureau of Ships at Davidson Laboratory demonstrate that

a plot of pitch or heave amplitude for a given wave height versus the ratio λ/L peaks or tends to level off at a ratio of λ/L between 1.8 and 2.4. Without the use of control surfaces, the path of a properly ballasted submarine will lock like figure 11 in a seaway where λ/L is 2.0.

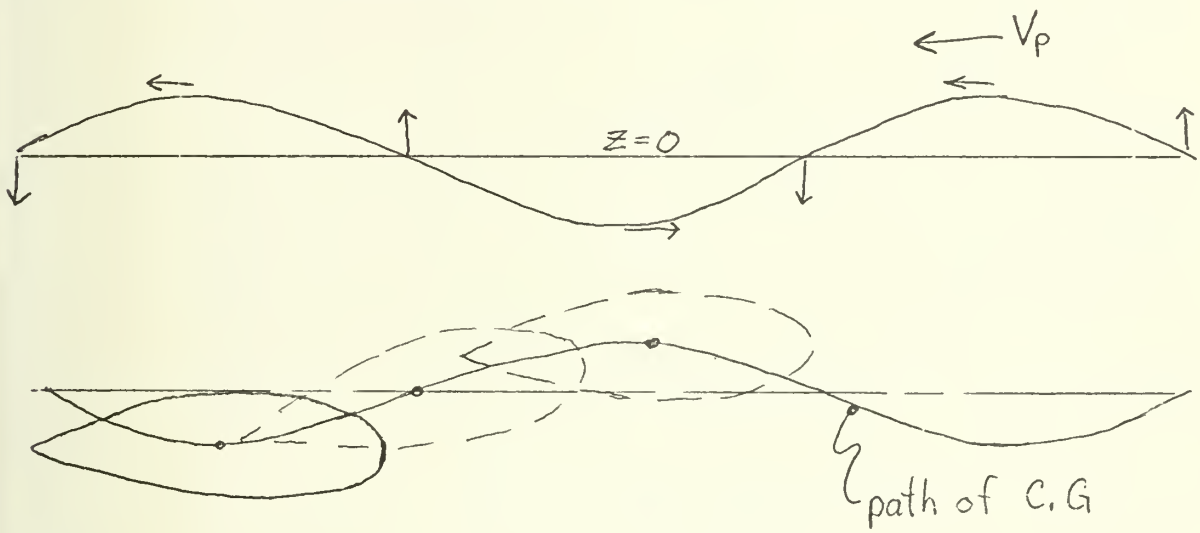


Figure 11

To counter this oscillatory path, the control surfaces must be used. Here again, the orbital velocity can be very significant. Although the effect is probably somewhat distorted in the case of X or cruciform stern planes which are generally shielded by the submarine's wake and the propeller race, the effect on the sail planes which are in the free stream can be critical.

For example, the lift generated by most control surfaces is proportional to the inflow angle - at least for small angles of attack. If a submarine's sail planes are at a zero angle of

attack and they encounter the trailing edge of a large wave at the center of its orbit, the new resulting angle of attack would create considerable lift. Note the following example:

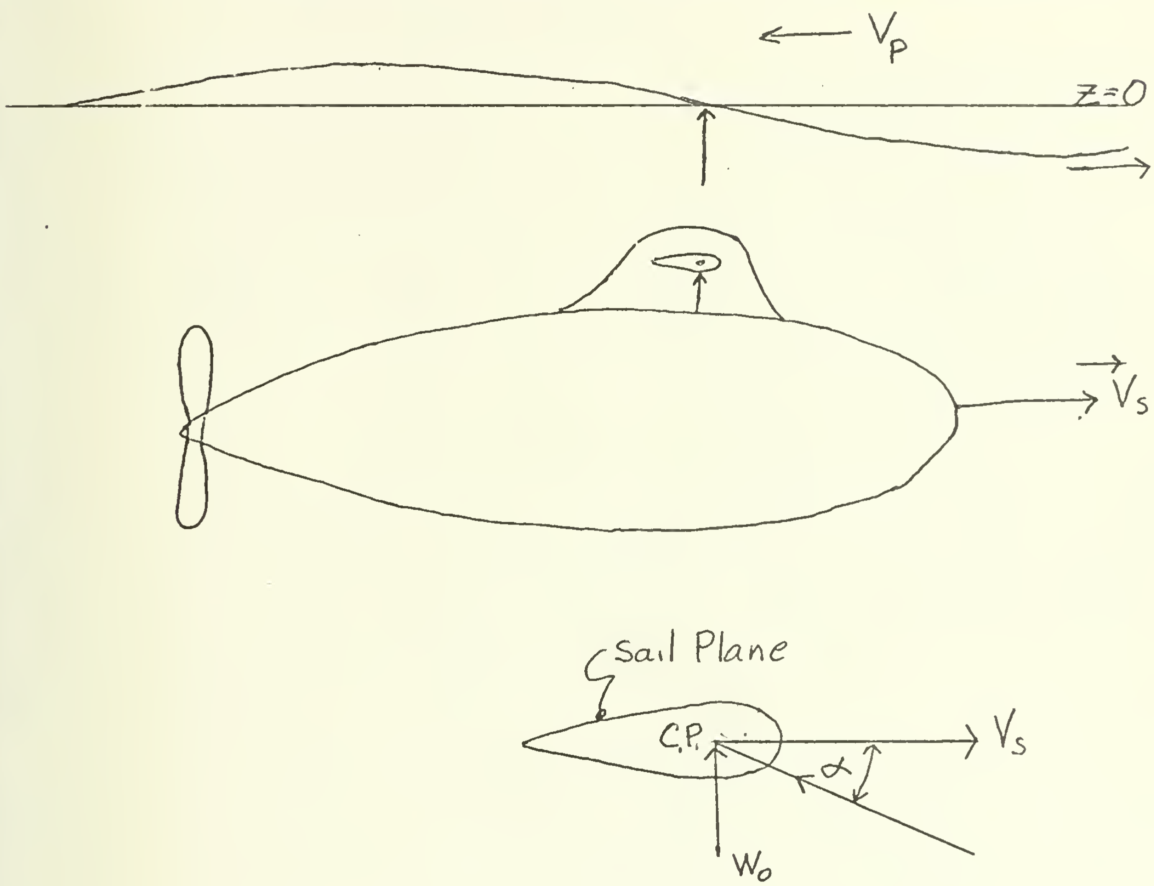


Figure 12

For a submarine at a velocity of 10 ft/sec and a depth of 55 ft under a 628 ft long wave with a 10 ft wave height,

$$z_{sail} \cong 30 \text{ ft}$$
$$\omega = \sqrt{2\pi g/\lambda} \cong .568 \text{ sec}^{-1}$$
$$W_0 = 0.568 \times 5 \times \exp[-0.3] \cong 2.1 \text{ ft/sec}$$
$$\alpha = \arctan\left(\frac{W_0}{V_s}\right) = \arctan\left(\frac{2.1}{10}\right)$$
$$\alpha \cong 11.8^\circ$$

This is not an extreme example. The problem becomes

particularly critical when for one reason or the other, the planes are already near their stall angle. Such an increase in the angle of attack here can cause loss of lift resulting in broaching or at least strong heaving.

Another hydrodynamic problem often occurs when skippers take on excessive amounts of ballast to offset the suck mentioned earlier. This normally results in a boat which is heavy by the stern. Typical angles are as much as 4 or 5 degrees down by the stern. This angle causes the boat to develop an angle of attack. Even for a slender body of revolution, this angle of attack will result in a net lift force on the entire body. This force must be countered in some manner or the body will rise to the surface. If the control planes are used to counter suck, control authority is compromised.

Previous discussions have concentrated on the effects of water particle orbital velocity. There is still one additional term from the pressure equation which must be considered when dealing with non-steady flow and that is the partial derivative with respect to time of the velocity potential, $\frac{\partial \phi}{\partial t}$. From the presentation of waves it is known that the velocity potential,

$$\phi = \frac{gh}{2\omega} \cos(Kx - \omega t + \epsilon)$$

for deep water. When considering a single sinusoidal wave, is purely arbitrary and may be dropped. Differentiation yields

$$\frac{\partial \phi}{\partial t} = \frac{gh}{2} \sin(Kx - \omega t) \qquad (32)$$

This term is seen to be linearly proportional to the wave height. A look at the following figure results in the corresponding

table comparing $\phi, \frac{\partial\phi}{\partial t}$, Z, and M.

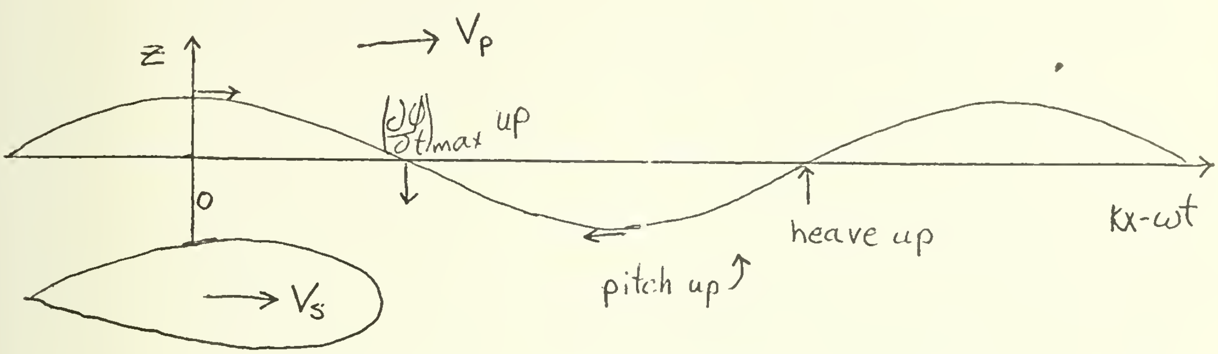


Figure 13

$$\left.\begin{array}{ll} \phi = \frac{gh}{2\omega}\cos(kx-\omega t) & \text{maximum at crest} \\ \frac{\partial\phi}{\partial t} = \frac{gh}{2}\sin(kx-\omega t) & \text{maximum at leading m.w.l.} \\ Z_o = -Z_{max}\sin(kx-\omega t) \propto h^2 & \text{maximum at trailing m.w.l.} \\ M_o = -M_{max}\cos(kx-\omega t) \propto h^2 & \text{maximum at trough} \end{array}\right\} \quad (33)$$

The $\frac{\partial\phi}{\partial t}$ term is out of phase with the orbital velocity of the particles and therefore tends to reduce the effect of the particle induced pitch and heave. Although the particle velocity can be determined if the height of the wave is known, it is considerable more difficult to determine the value of $\frac{\partial\phi}{\partial t}$ in a random sea. In order to that the entire spectrum must be considered and a complicated analysis is required involving the differentiation of an infinite sum of unknown velocity potentials of different magnitudes, frequencies, and phases. Since the pitch and heave, M_o and Z_o, are proportional to the wave height squared, and the $\frac{\partial\phi}{\partial t}$ term is only proportional to the first order of height, the $\frac{\partial\phi}{\partial t}$ term is usually neglected.

When discussing Venturi's effect and the orbital velocity of the water particles, it was mentioned that not only did the free surface and its waves affect the motions of the body but that the motions of the body altered the configuration of the surface and the path of the particles. It is permissible to ignore the effects of the body on the environment for the purposes of initial analysis. However, if truly meaningful results are to be obtained for use in a control system, it is necessary to consider these parametric excitations. Unfortunately even though the environment's excitation of the body and even the body's coupling between its own motions can be predicted with reasonable accuracy in the normal homogeneous equations of motion, the body's effect on the environment can be of the fourth order and requires extensive analog simulation.

An overall survey of the excitations experienced by the near-surface submarine reveals that they may be broken into two basic catagories. The first is a d.c. component in the form of the suck, and the second is an a.c. component which is a combination of the various oscillatory forces. Both the d.c. and a.c. effects in a regular wave are primarily dependent on second order terms. When the body is subjected to a stochastic seaway, it is then understandable that accurate simulation and prediction becomes quite involved if not impossible.

Control Systems

The excitations mentioned in the preceding section present a variety of control problems and even a wider range of potential solutions. Before the actual selection and design of the control system can begin, a list of objectives and constraints must be assembled as guidelines for the design.

First and foremost, the entire system must be stable. Without using the control surfaces a near-surface submarine unstable in heave. The first thing that the submarine must do then is counter this destabilizing force.

A second objective is accuracy. This is best understood by noticing the difference between the output and the command input of the system. Nearly all control systems have some steady state error after the transient response dies out. Accuracy calls for a minimization of this error. The transient and steady state response are indicated in figure 14 for a second order system subject to a unit step input.

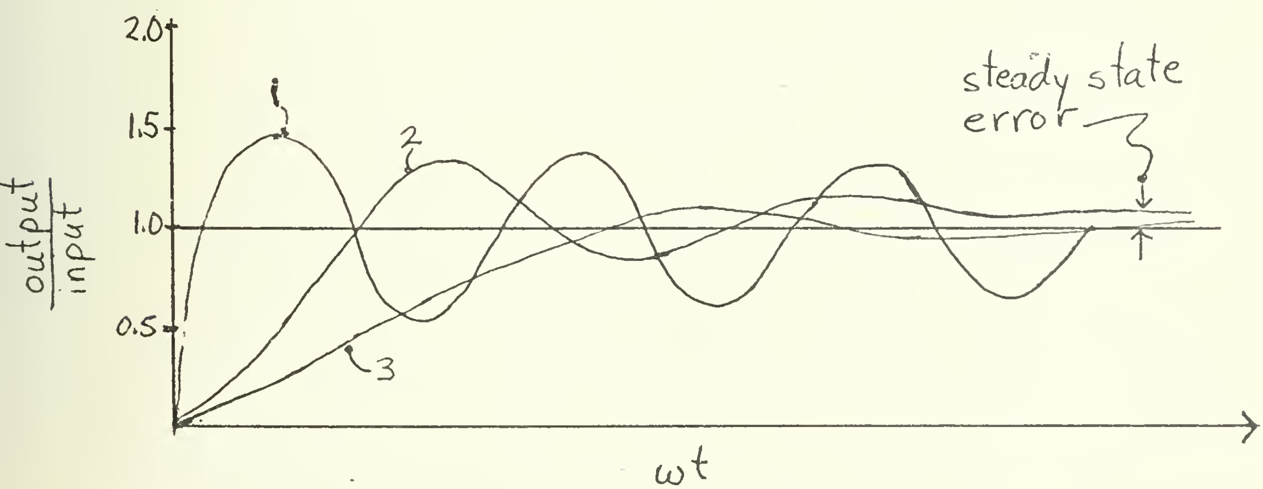


Figure 14

Although a second order system is usually inherently stable, a near-surface submarine is assuredly a high order system. Nevertheless, the second order graph in the figure is still valuable in comprehending various properties of the system. In a field of excitation such as a random ocean in a high sea state, speed of response is also very important. The system umst react quickly to each significant excitation before the problem is further complicated by additional excitations. If the response is too slow, the submarine will soon succumb to the rapidly changing environment.

A fast response is however not in itself sufficient. Curve 1 has a very fast response but it takes an extremely long time to settle into a steady state. Such oscillation in a submarine would prove to be very uncomfortable and the continuous superposition of more oscillations would soon create a dangerous situation. For these reasons the system should be equipped with a fair amount of dampening.

Speed of response and stability are both generally aided by using some type of proportional control with a high gain. In other words, the control surfaces are deflected a given amount per unit heave or pitch. The more they are deflected per unit heave or pitch, the faster the corresponding force will act. (Care must be taken even here because too high of a gain can hurt stability and it certainly hurts the dampening of the system.)

Derivative control is generally used to insure proper

dampening. This requires that the rate of heave, $\dot{z}$, or pitch, $\dot{\theta}$, be detected. The surfaces are then deflected in accordance with rate as well as amplitude of motion. In general then the angle of deflection of a surface is,

$$\delta(t) = K_1 z(t) + K_2 \dot{z}(t) \qquad (34)$$

where K_1, and K_2 are the gains, z is the heave amplitude, and $\dot{z}$ is the heave velocity.

A quality which relates the steady state error to the excitation from the environment is known as stiffness. Stiffness is defined as the ratio of the steady state error to the amplitude of the excitation. For a submarine in a random sea, this is:

$$S = \overline{\left[Z_{command} - Z_{output} \right]^2} / \overline{h}^2 \qquad (35)$$

Noise, or lack thereof, is another mark of a good control system. In general noise occurs when very high gains are employed and it is one of the major deterrents in trying to optimize the control system. It also sometimes results when the amplitude and rate of the ordered deflection approaches or reaches the maximum capabilities of the actuators. Noise in the electronics of the control system can result in substantial losses of accuracy and speed.

Control authority has already been mentioned in a previous section, but again care must be taken to avoid too much compromise in control authority.

Finally life cycle cost must be foremost in any designer's mind. Life cycle cost is used here because acquisition cost is generally an almost insignificant quantity when compared to

research and development costs not to mention repair and maintenance costs. In fact, cost is probably the primary obstacle in the path to the development of improved control systems.

There are basically two classifications of control systems, closed loop or feedback control and open loop. Open loop systems are those in which the input or command signal is totally unaffected by the output and its resulting error. In a closed loop system, the error at the output is fed back to the controller to help reduce the error. Note the following simplified block diagrams of both systems.[11]

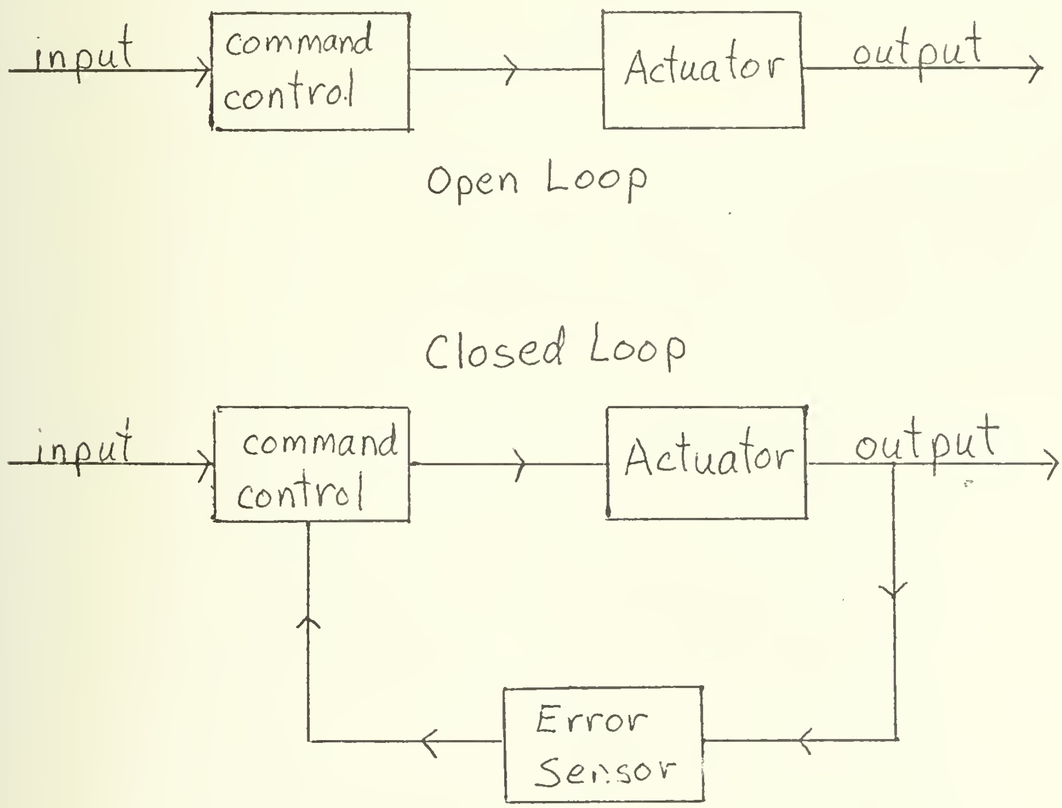


Figure 15

If the controller knew precisely how to react for each

input, he then could properly control the actuators (control surfaces) to result in zero error. Needless to say, under a random ocean, it is quite impossible to perfectly predict the excitation. To further complicate the situation, the equations of motion are only approximations of the vehicle response to its own actuators in a regular sea. Obviously then a feedback system is essential for the control of a submarine.

The following sections will discuss the closed loop control system now in use as well as three other possible alternatives, the intent being to optimize as many of the preceding objectives as possible.

Manual Control (#1)

Before deciding what type of control system is to be used in a new submarine, it is advisable to examine existing systems. The primary differences among today's existing military submarines which affect constant velocity control near the surface are various external configurations. These differences include size and location of the sail, location of forward control surfaces (bow or sail), and location and arrangement of after control surfaces. Despite these apparent differences, the control portions of these systems are relatively similar and will therefore be discussed collectively. Exceptions to this statement are some experimental and special purpose boats such as the DSRV.)

The block diagram in figure 16 is a simple but representative example of such systems. An order to come to and maintain a certain depth at a given attitude (pitch andle) is given by the Officer of the Deck. This order is the input. The two planesmen and the Diving Officer then work closely together to achieve the desired output. First the Diving Officer makes the decision to take on extra ballast (beyond that required for neutral buoyancy). The amount he orders is generally a function of what knowledge he may have about the existing sea state, his previous experiences, and/or formal boat operating procedures. He periodically corrects this decision based upon his observation of the average angle of attack required by the sail planesman to maintain depth. In other words, if the plane

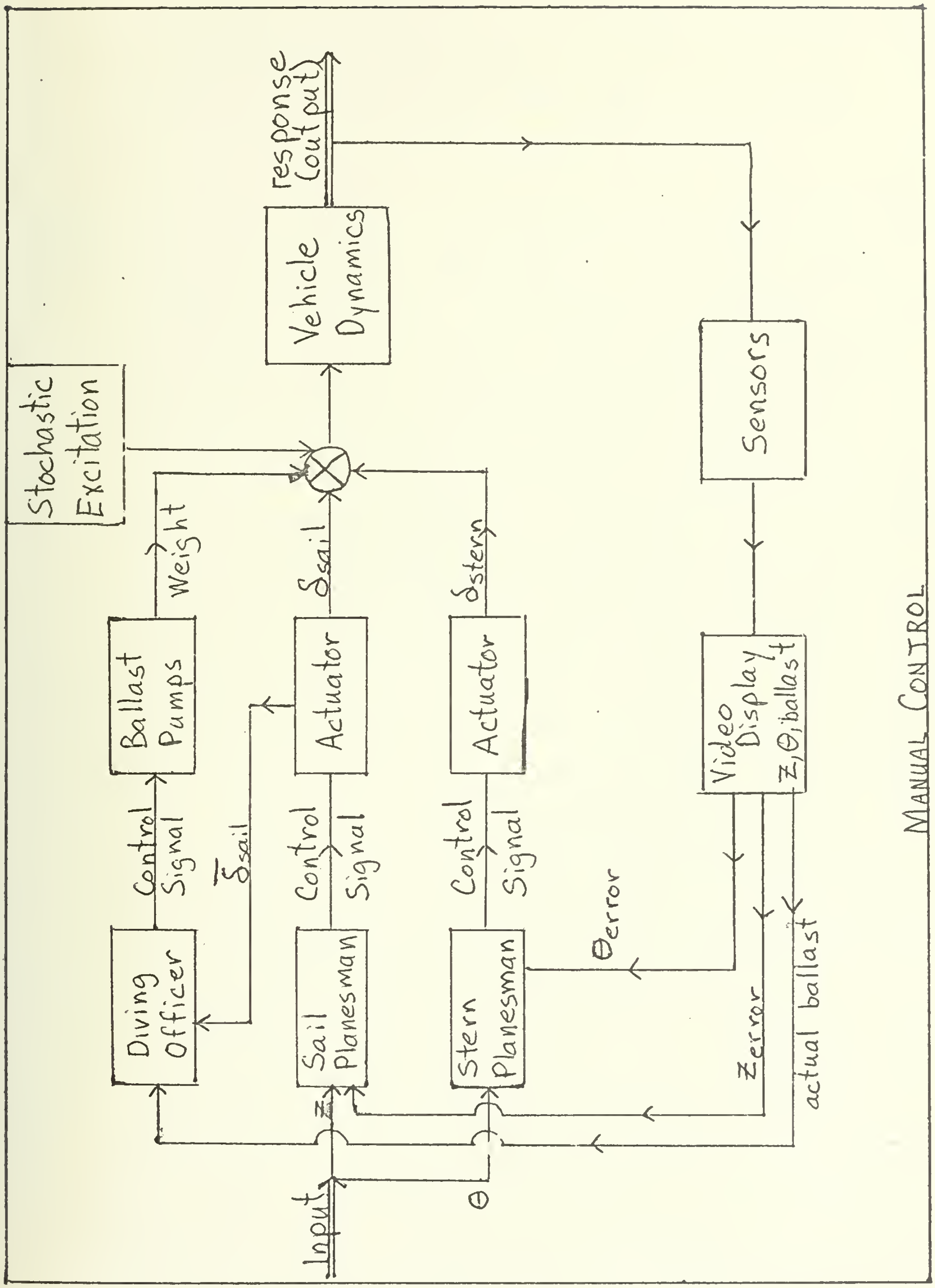

Input
response (output)
Stochastic Excitation
Vehicle Dynamics
Sensors
Video Display
z, θ, ballast
Diving Officer
Ballast Pumps
Control Signal
Weight
Sail Planesman
Actuator
Control Signal
δ_{sail}
$\bar{\delta}_{sail}$
Stern Planesman
Actuator
Control Signal
δ_{stern}
θ_{error}
z_{error}
actual ballast
z
θ
MANUAL CONTROL

Figure 16

averages an up angle of deflection of five degrees for instance, he knows that the plane is being used to counter the suck force as well as oscillatory heave. He would then take on more ballast.

The stern planesman and the sail planesman work as a team. The ~~stern~~ sail planesman follows a gage which is continually displaying the depth of the boat. He deflects his planes in accordance with the amount of error between the actual and command depth and how fast he senses that this error is changing. Similarly, the stern plane operator follows a bubble which continuously shows the angle of pitch. The justification for this separation of control stems primarily from plane locations. Since the sail plane is located relatively near the center of buoyancy, the moment arm of its vertical force will be relatively small and therefore its capability for inducing pitch is relatively small. On the other hand, the stern planes are placed as far from the center of buoyancy as reasonably possible. Of course their pitching moment is accompanied by a heave force, but since the moment arm is so lang, the magnitude of the force remains small enough to be corrected by the sail plane.

The actions of the planesmen are carried by electric signals to the hydraulic system which actuates the respective planes. The resulting motion is a direct result of the control surface deflections, the ballast condition, and the exciting forces of the environment. This motion is monitored and the difference between the output and input (error) is displayed for

the planesmen in the form of the readings on their gages. Men close the control loop mentioned in this section when they visually note the difference in the actual depth or pitch angle and the commanded position and act to correct it.

The advantages of this system are readily apparent. It is the simplest system available that will get the job done. Since it has been proven in actual use for many years, further research and development costs for installation into future boats are relatively insignificant.

Maintenance is low since men are the decision makers and thus the amount of electronic gear is minimized. The required number of sensors is small and they are relatively simple. The pitch sensor is a gravity device and the depth sensor measures hydrostatic pressure. Another feature associated with this low maintenance and simplicity is a high degree of operability. The hydraulic system which controls the planes and the simple electric circuit which transmits the orders from the planesmen to the hydraulics system are both very reliable. Off duty planesmen are always available in the event of sickness or injury to one of the planesmen.

Another perhaps intangible benefit but nonetheless significant is the fact that most skippers tend to distrust black boxes. This is really not so hard to comprehend considering the Navy's tendency to outfit operational combatant ships with "new, improved" hardware based on computer simulations and small scale model tests rather than full scale testing. Such experi-

mental procedures can be justified on a time-cost basis but few skippers like the idea of risking their records for the sake of experimental research.

A very tangible and tactically rewarding benefit to this type of control system is its low degree of detectability. It does not rely on any forward search sonar to predict effects of oncoming waves. This very beneficial when operating in close proximity to enemy forces.

There are reasons however why many operators and designers wish to improve control systems. They point to a number of "faults" in the current system. Looking first at the stability, it is obvious that problem does indeed exist. This has in many cases resulted in broaching. The manner in which the Diving Officer chooses the amount of extra ballast to counter the d.c. effecto of various sea states is something less than ideal. Without precise knowledge of current surface conditions, this officer must make a calculated first guess and then make corrections as he gains more information (average deflection of sail planes amd relative roughness of ride). Since the most crucial time in a near-surface cruise is the ascent from deep depth and the initial leveling off, a more knowledgeable officer would result in fewer broachings. Additionally while many officers may know the proportionality constant between lift and the angle of attack, they must rely on "eyeball averages" when noting average plane deflections. This can lead to inaccuracies which can only be corrected by trial and error. In

fact, the optimum ballast condition many times is never reached for an entire near-surface mission.

Consider the case for a submarine at a given constant speed. Recalling that ballast is taken on to counter suck, the time averaged lift force is desired. The oscillatory forces will average to zero and only the lift necessary to counter suck will be left.

$$L(t) = L_{suck} + L_{oscillatory} \tag{36}$$

where L_{suck} = Suck Force - Extra Ballast (37)

and $L_{oscillatory}$ = Lift due to sinusoidal components of orbital velocities (38)

$$\overline{L(t)} = L_{suck} + 0 \tag{39}$$

$$L_{suck} = \frac{1}{2}\rho A V^2 C_{\alpha}(\alpha)$$

$$\text{where } C(\alpha) = C_0 + C_{\alpha}\overline{\alpha} \tag{40}$$

For a symmetrical foil or plane, $C_0=0$

$$L_{suck} = \frac{1}{2}\rho A V^2 C_{\alpha}\overline{\alpha}$$

$$\text{and } \alpha = \delta_{sail\ plane} + \arctan\left[\frac{W_0(t)}{V+u_0(t)}\right] \tag{41}$$

Averaging over time results in the second term on the right side being zero so,

$$\overline{\alpha} = \overline{\delta} \tag{42}$$

Now it can be seen that

$$L_{suck} = \frac{1}{2}\rho A V^2 C_{\alpha}\overline{\delta} \tag{43}$$

$\overline{\delta}$ is the only questionable number in this equation. If $\overline{\delta}$ is known, L_{suck} can be precisely calculated. L_{suck} is the amount of excess ballast or additional ballast needed depending upon its sign.

Due to the criticality of having enough ballast the Diving Officer often takes on more than necessary. This requires use of the sail planes to keep the vessel from sinking. Since most of the ballast is generally aft of the center of buoyancy, it also causes the bow to pitch up. This results in the aforementioned angle of attack on the entire submarine and an additional lift in the same direction as the suck force. This requires countering with more ballast or additional use of control surfaces and a corresponding decrease in control authority. Such measures and countermeasures can have a very destabilizing effect on a submarine.

Another critical variable in successful control in this stochastic environment is the planesmen. Even with perfect control of ballast, accuracy depends on the skill, composure, and coordination of the two planesmen. Under duress, it is conceivable that even the most professional planesmen's performances would suffer. Another feature highly dependent upon the "human factor" is speed. Operation becomes hazardous at speeds above a certain critical speed. Above this speed it can no longer be assumed that a man will consistently react quickly enough to even normal excitations to avert mishaps - broaching in particular. This "reaction time" is different for each man and is obviously quite difficult to measure with any real degree of certainty. For this reason, submarine operating procedures restrict speed near the surface to arbitrarily low numbers. If more accurate modeling were possible, it is probable

that upper limits would be raised. This inpredictability is another shortcoming of this type of system.

The mandatory speed reduction carries with it some mixed blessings. Besides insuring that speeds ard kept within human reaction limits, it limits if not eliminates noisy and inefficient cavitation since cavitation is dependent upon the squared angular velocity of the propeller blades. Low speeds limit the effect of the boat's own velocity on the d. c. suction force. Unfortunately, low speeds also decrease the lift capabilities of the control surfaces. Remember, $L=\frac{1}{2}\rho A C_L \underline{V^2}$. They also causethe oscillatory forces and moments due to water particle velocities to play a larger role. This is particularly true for the case of the control surfaces. Again recall that lift is proportional to the angle of attack and that the angle of attack depends on the ratio of the vertical component of the orbital velocity to the relative horizontal velocity. For the extreme limiting case of a submarine with zero speed,

$$\alpha = \delta + \arctan\left[w/(V+u)\right]$$

$$\alpha = \delta + \arctan\left[\frac{W_{max}\sin\omega t}{U_{max}\cos\omega t}\right] = \delta + \omega t \qquad (44)$$

As long as the control loop does not have a look ahead or anticipatory facility, there will always be a time lag due to the time required for a disturbance to be sensed, displayed, reacted to, an order given, a signal transmitted, and the control surfaces actuated. For these reasons the system as it is can never give a "perfect" ride. For the same physical characteristics and assuming that each component is operating at

maximum efficiency, the system which minimizes or eliminates this time lag will be the one with the potential to become most efficient and comfortable.

The system has certainly contributed its share to successful submarine performance in the past and will probably continue to do so in atleast the near future. At this time in fact it even appears to be the best solution to the problem of near surface control if commercial submarines are ever used for cargo transportation.

Computerized Reaction Control (#2)

The most logical method to consider next for improving the existing system is to replace the man in the loop with a machine which will react more quickly to the sensor input.

The process flows around the loop in the following manner. (See figure 17) The Officer of the Deck gives the commanded depth and pitch angle, the Diving Officer selects his ballast much as he did in the first case (His primary job is to offset the d.c. force and so does not enter into the lag time.), and the control surfaces are deflected in accordance with some preset formula of the following type.

$$\delta_{sail} = K_1[Z_o - Z_{command}] + K_2 \dot{Z}$$
$$\delta_{stern} = K_3[\theta_o - \theta_{command}] + K_4 \dot{\theta} \qquad (45)$$

where K_1, K_2, K_3, and K_4 are constants chosen from experimental and classical analysis.

The planes are deflected accordingly. The environment, the ballast condition, and control surface attitudes result in various dynamic forces and moments which cause the submarine to respond and assume some new depth and pitch angle. The differences between the two outputs and inputs are sensed and sent as input to the black box. Simultaneously θ and z are visually displayed for monitoring by the Officer of the Deck and the Diving Officer who continues to correct his initial selection of ballast.

The black box changes the physical set up in a number of ways. It eliminates the need for the control knobs used by

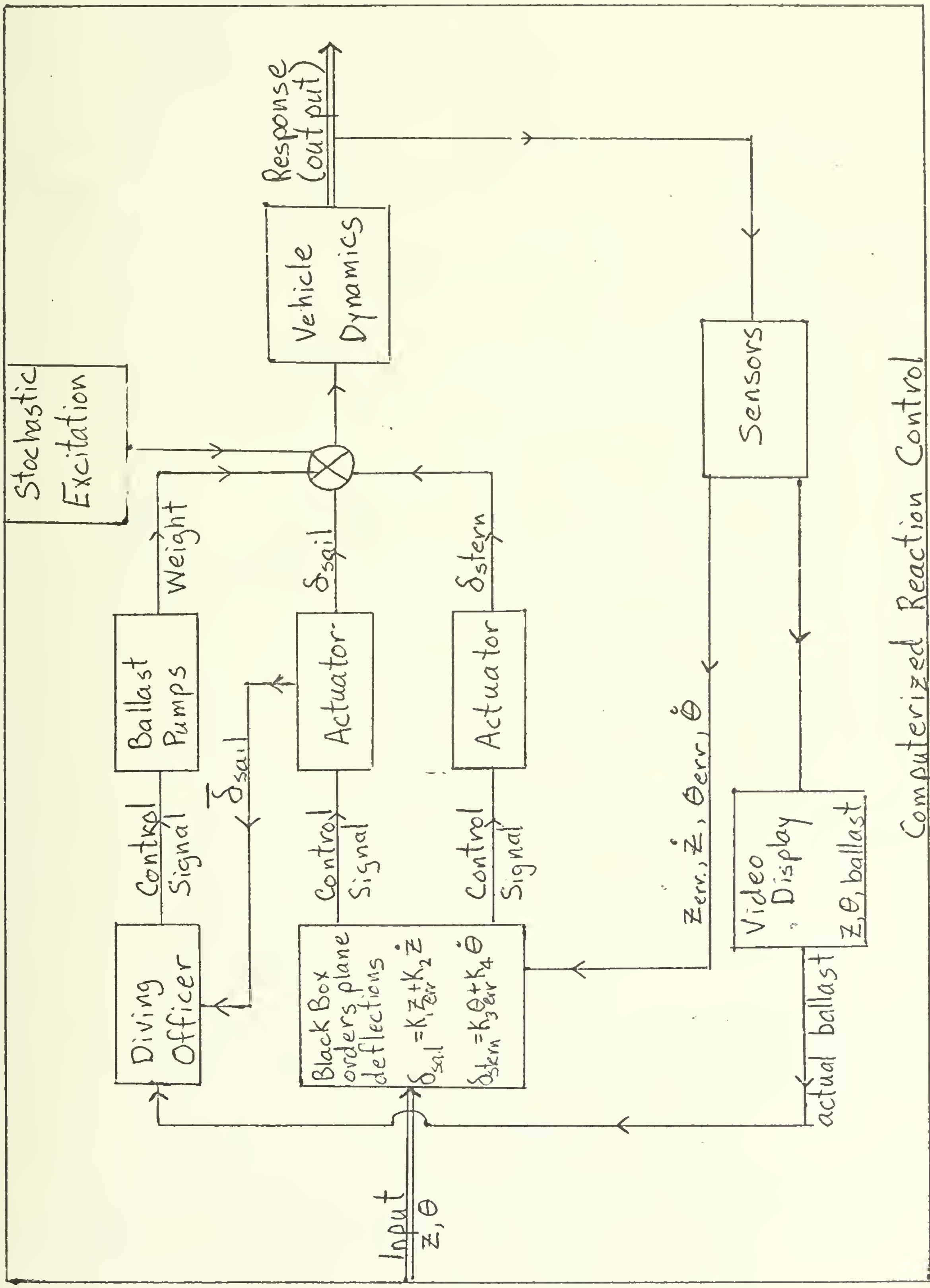

Stochastic Excitation
Vehicle Dynamics
Response (output)
Sensors
Ballast Pumps
Weight
Actuator
δ_{sail}
Actuator
δ_{stern}
$\bar{\delta}_{sail}$
Control Signal
Control Signal
Control Signal
Diving Officer
Black Box orders plane deflections
$\delta_{sail} = K_1 z_{err} + K_2 \dot{z}$
$\delta_{stern} = K_3 \theta_{err} + K_4 \dot{\theta}$
$z_{err}, \dot{z}, \theta_{err}, \dot{\theta}$
Video Display
z, θ, ballast
actual ballast
Input
z, θ
Computerized Reaction Control

Figure 17

the planesmen - this saves time. It also eliminates the display time from the oscillatory control loop since the black box receives the error directly from the sensors. Effectively then the display unit, the decision unit, and the control signal unit are included in one neat box.

This system offers several advantages over the original manual system. The elimination of display, human decision making, and manual turning of wheels reduces reaction time considerably which keeps forces and moments from building up.

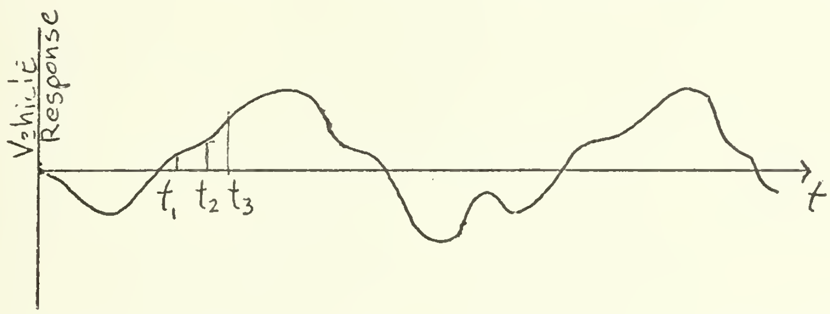


Figure 18

t_1 error sensed

t_2 automatic system reacts and sends control signal to hydraulic actuators

t_3 manual system reacts and sends control signal to hydraulic actuators

This reduction in reaction time will allow higher speed and the automation will allow for uniformity of performance from watch to watch and boat to boat.

The problem of choosing the constants in equation (45) is no small talk, but it is reasonable to assume that they can be chosen to at least equal the best human performance if by no

other way than using the same proportionality constants a good planesman uses (subconscious though his actions may be). Once this process reaches a suitable solution, a decrease in error should result in a more efficient system.

NAV SEC has conducted tests in which it compared the error of the simulated system with a man in the loop to a similar system which has replaced the man. The simulation showed that the automatic system had in all cases less than one half the error of the manual system. The automatic system was not only much quicker with its response, but it was much more sensitive to excitations. In simulation, the results were very encouraging. Unfortunately though, in the process of achieving this apparent break through, the simulated control orders exceeded the capabilities of existing hardware.

The obvious solution to this problem is to install some filters so the less significant excitations are ignored and some other filters to limit the ordered deflection rates and/or amplitudes. This of course negates part of the advantage which has been gained. It also makes the system as a whole and thus modeling, repair, and maintenance more complicated and less reliable.

As far as today's Navy is concerned, perhaps the biggest advantage of such a system is that it would release at least two men and reduce the training requirements for three others. (On most boats, one of the planesmen operates the helm as well as his plane. For a three section crew requiring a total of

six planesmen, three planesmen could be released and the other three would only have to be profficient at the helm. One additional man would probably be needed for maintenance and repair of the computer.) Over the life of a vessel, this reduction results in the savings of hundreds of thousands of dollars.

The problems accompanying the suck force are essentially identical to those of the manual method since this portion of the loop is unchanged.

From a combatant standpoint, a problem arises in regard to operability and survivability of the system when attacked. A depth charge is certainly more likely to decommission a machine than kill or otherwise incapacitate a man. In the long term perspective, the problem can be solved by having a manually controlled back up system much like the previous system. The solution to the transient problem is not quite so simple. What is to happen to the boat between the time the automatic system fails and the time manual control is resumed? Will the submarine broach, will it go into a crash dive, or what will it do? This of course depends upon a number of things - type of failure, depth at time, ballast condition, control surface situation, etc. Of course someone could be on hand at all times as a back up to operate the manual controls in case of a mishap. This would compromise the advantage gained by reducing the manning requirements.

It now becomes apparent that this type of system has

restrictions that can not be overcome by the design engineer. The amount he can reduce reaction time will always be dependent upon technology and thus beyond his immediate control. Because the rate of deflection of the control surfaces is finite, this type of system will always have a lag time between the input and the output.

Anticipatory Automatic Control (#3)

It was evident in the previous section that if further improvements were to occur, a way must be found to overcome the inevitable time lag. Since it could not be eliminated, the best bet seems to be to compensate for it. This will require an anticipatory system. In otherwords, there must be some means of accurately predicting the oncoming excitations before they reach the boat. The control loop for such a system is pictured in figure 19.

This loop differs substantially from the first two. In addition to the normal depth and pitch input, wave characteristics are also inputs. From the summing junction, either a classical solution involving the equations of motion or a computerized strip analysis could be used to determine how much and how fast the control surfaces should be deflected. If either the equations of motion or the strip theory could produce exact solutions (rather than approximations) no feedback loop would be necessary. Obviously though such is not the case and additional input to the summing junction is the error of the system via the feedback loop. The ballast is another feedback which is also an input to the summing junction. The plane deflections are monitored and averaged by another computer which calculates ballast requirements and continuously varies the ballast accordingly.

The beauty of this type of system is that the planes will be moving continuously in anticipation of what comes next.

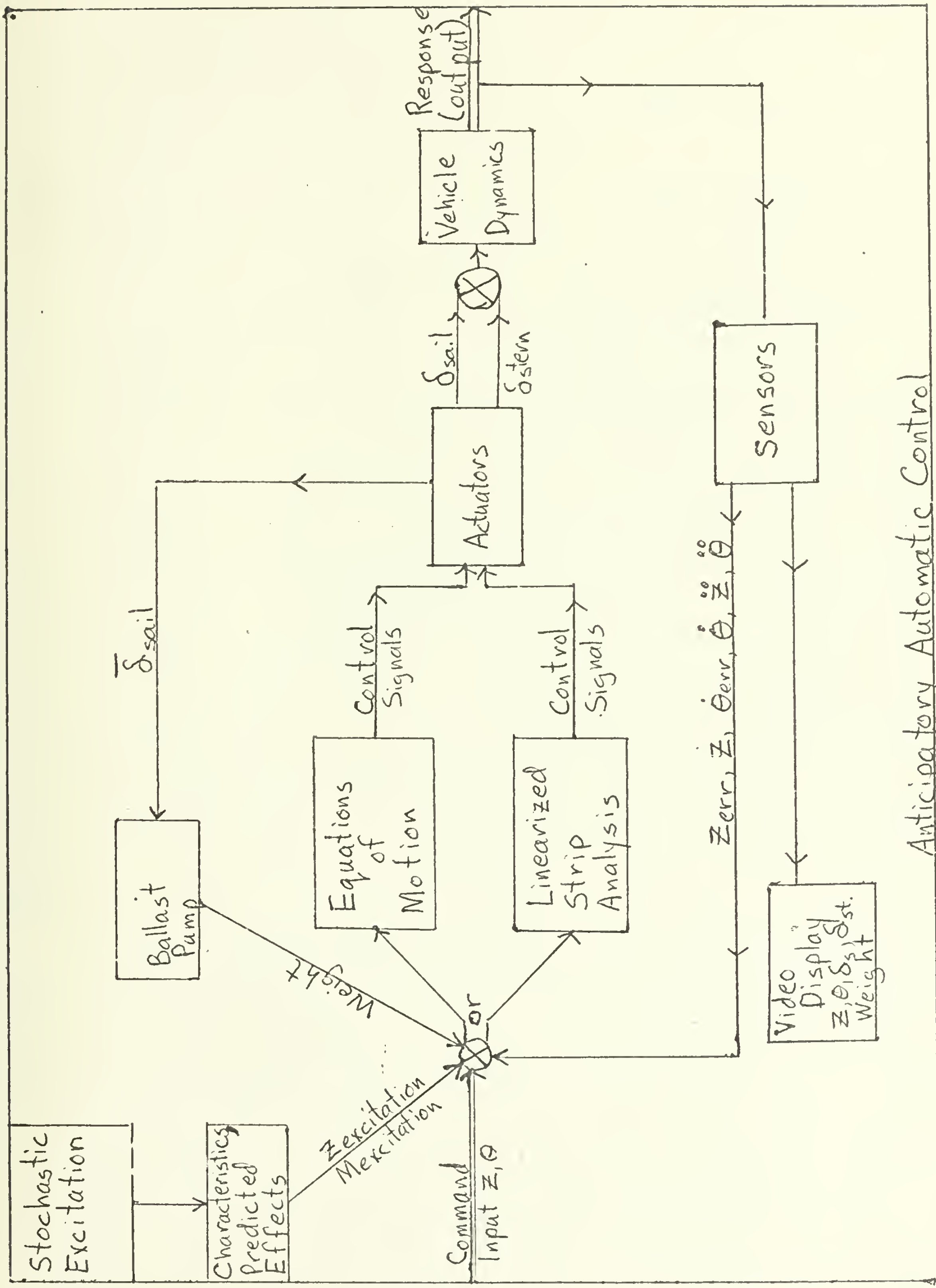

Stochastic Excitation
Characteristics Predicted Effects
Zexcitation
Mexcitation
Command Input z,θ
Ballast Pump
Weight
$\bar{\delta}_{sail}$
Equations of Motion
Linearized Strip Analysis
Control Signals
Control Signals
or
Actuators
δ_{sail}
δ_{stern}
Vehicle Dynamics
Response (output)
Sensors
$Z_{err}, Z, \theta_{err}, \dot{\theta}, \theta, \ddot{Z}, \ddot{\theta}$
Video Display $z, \theta, \delta_s, \delta_{st}$, Weight
Anticipatory Automatic Control

Figure 19

Although the planes move continuously, the path of the boat is always horizontal (theoretically). The ability of the system to predict the wave and its effect on the boat will be one of the deciding factors for accurate control.

The output of the wave prediction subsystem will depend upon which type of solution is to be used, classical or strip theory. In both cases however it is necessary for the wave characteristics to be determined in advance.

Displays of $z, \Theta, \delta_{sail}, \delta_{stern}$ are for monitoring only and serve no other purpose as long as the system is functioning properly.

This type of control has many tangible advantages. The reduced manning requirements would be similar to those of the previous method but the Diving Officer would now be free to devote his entire attention to other duties. The Officer of the Deck will not have to worry about bleary-eyed planesmen. Now that a computer has replaced the Diving Officer at the ballast controls, the weight is not subject to human miscalculations.

The biggest advantage of course is that the anticipatory capability makes a perfect ride theoretically possible. Even though it is obvious after noting the approximations used in both the classical and strip theory that this will not happen, control probably can be improved by at least an order of magnitude if enough time and money are spent on modeling and simulation. Speed will no longer be limited by system lags. (Though in most cases propeller cavitation probably will limit

it.) Finally, because of the anticipatory nature of the system, less energy will be wasted in attempts to rapidly deflect the planes by large amounts to catch up with the rapidly changing environment. Instead necessary angles will be smaller and high deflection rates will be less imperative. ("An ounce of prevention...")

Unfortunately, the intricacies which give the system such high potential also leave it open for possible disaster. The accuracy of the sonar is subject to question due to the composition of the reflecting surface. The submarine's proximity to the surface could cause the receivers to pick up much unwanted noise which would be hard to differentiate from the desired back-scattering. The assumption of regular waves is necessary if things are to be workable, but the accuracy, especially when predicting particle velocities, is suspect. This is particularly true since determination of wave amplitude relies on the assumption that the submarine knows its exact depth below <u>mean</u> sealevel constantly.

A look at the classical method quickly eliminates it from consideration since the hydrodynamic coefficients become dependent on depth and sea state if the boat is within a half of a wave length of the surface. Notice the Z_z plot in Appendix 2. Decoupling the roll equation in other than head or following seas is very unrealistic. This is the parametric excitation mentioned earlier. Additionally it should be noted that the equations of motion are first order approximations but the suck

force is second order and therefore not included. An expansion of the equations of motion to include all second order terms is very complex. Even this would not insure accuracy. It would still be extremely difficult to model the equations.

Linear strip theory appears to be a more plausible solution. It still is subject to the wave prediction problems. It will account for beam seas and for roll to some extent. Problems in this method lie in prediction of boundary conditions. It is difficult to describe the influence of the water particle orbital velocities on the boundary layer. This particularly true on the side opposite the direction of the prevailing seas. Effects of separation near the stern and of the sail vortex on the afterbody are neglected.

Another problem expecially in enemy waters is control of the sonar pulse necessary to predict the waves. Its transmission could be in violation of normal audo silence. Even if everything else worked flawlessly, this would probably be enough to doom the system.

A back up crew will still be necessary in case of some mishap. This problem is identical to the previous system.

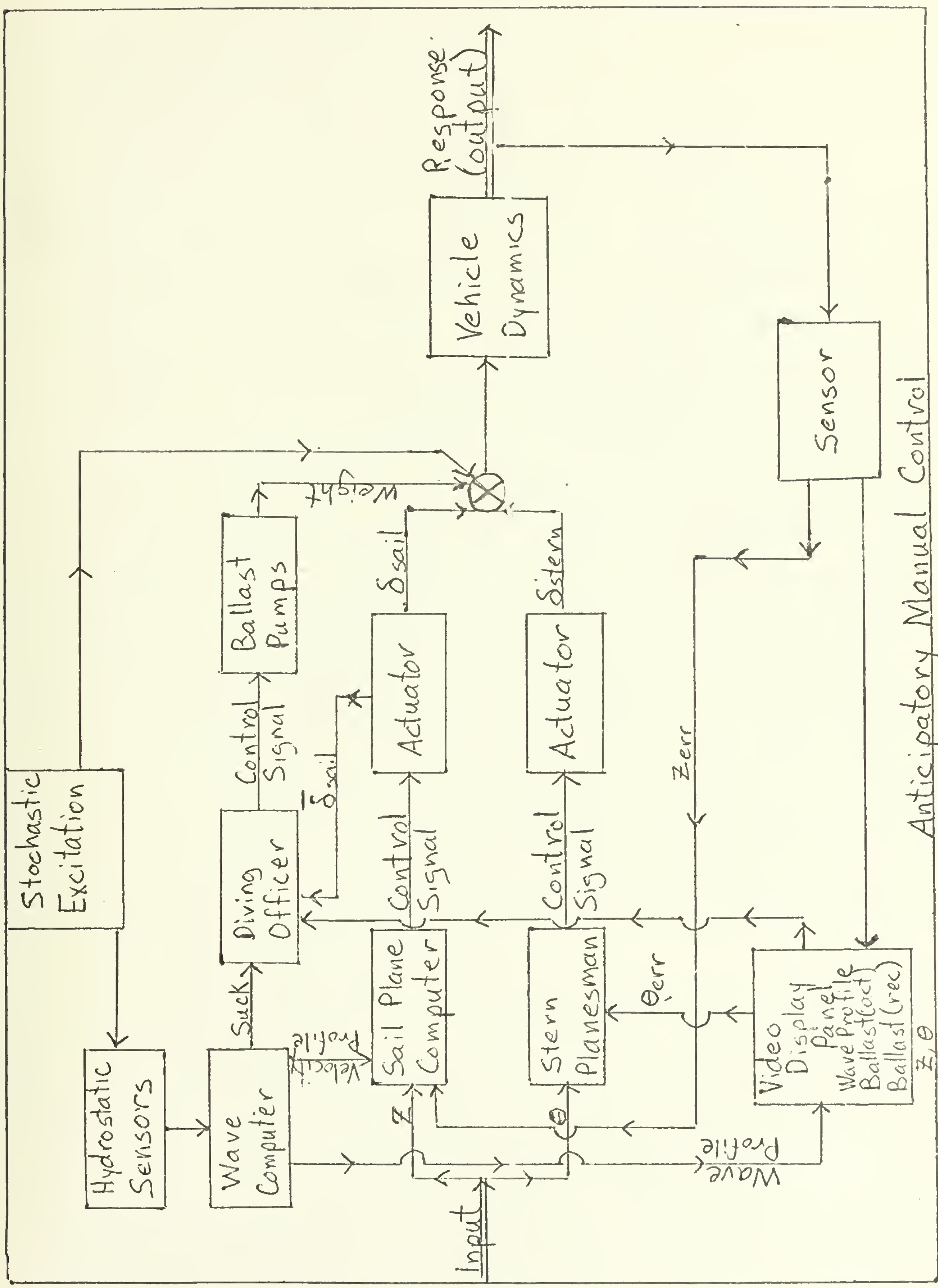
The final argument against this system would come from financial experts. Research and development costs would be astronomical, especially in the form of model testing and computer simulation. Even if these were successful, full scale tests would seem to be a must. Additionally, maintenance and repair costs would certainly be very restrictive.

Anticipatory Manual Control (#4)

There are of course numerous other conceivable systems. However, faced with the need for an anticipatory system and the reality that the previous automatic system is presently infeasible, the following is one of the solutions that presents itself. (See figure 20 for control loop.)

The depth input for this system goes to a computer which controls the sail plane. The pitch input goes to the stern planesman. Hydrostatic sensors along the top of the hull measure depth of the hull at that point. With this information, a reasonably accurate wave profile can be plotted within the computer. This serves two purposes. The water particle orbital velocities and directions can be determined for use by the sail plane. The angle of attack can be more accurately predicted. (In the first two systems steady uniform flow over the planes was assumed.) This anticiaption of angle of attack can make a significant difference and it will allow the planes to operate more efficiently - at lower angles of deflection.

The second use of the wave profile is as a display for the stern planesman. A computer does not replace the man in this loop because inflow velocity predictions are far too inaccurate for use. A combination of propeller race, afterbody separation, and sail vortices create the problem. Instead a man is given an anticipatory capability by giving him a video display of the encountered wave shape. He can easily be trained to anticipate how the submarine will act under various portions of variously



Stochastic Excitation
Hydrostatic Sensors
Wave Computer
Suck
Profile
Velocity
Diving Officer
Control Signal
Ballast Pumps
Weight
$\bar{\delta}$sail
Sail Plane Computer
Control Signal
Actuator
δsail
Stern Planesman
Control Signal
Actuator
δstern
Vehicle Dynamics
Response (output)
Sensor
Zerr
θerr
Video Display Panel
Wave Profile
Ballast (act)
Ballast (rec)
Z, θ
Z
θ
Wave Profile
Input
Anticipatory Manual Control

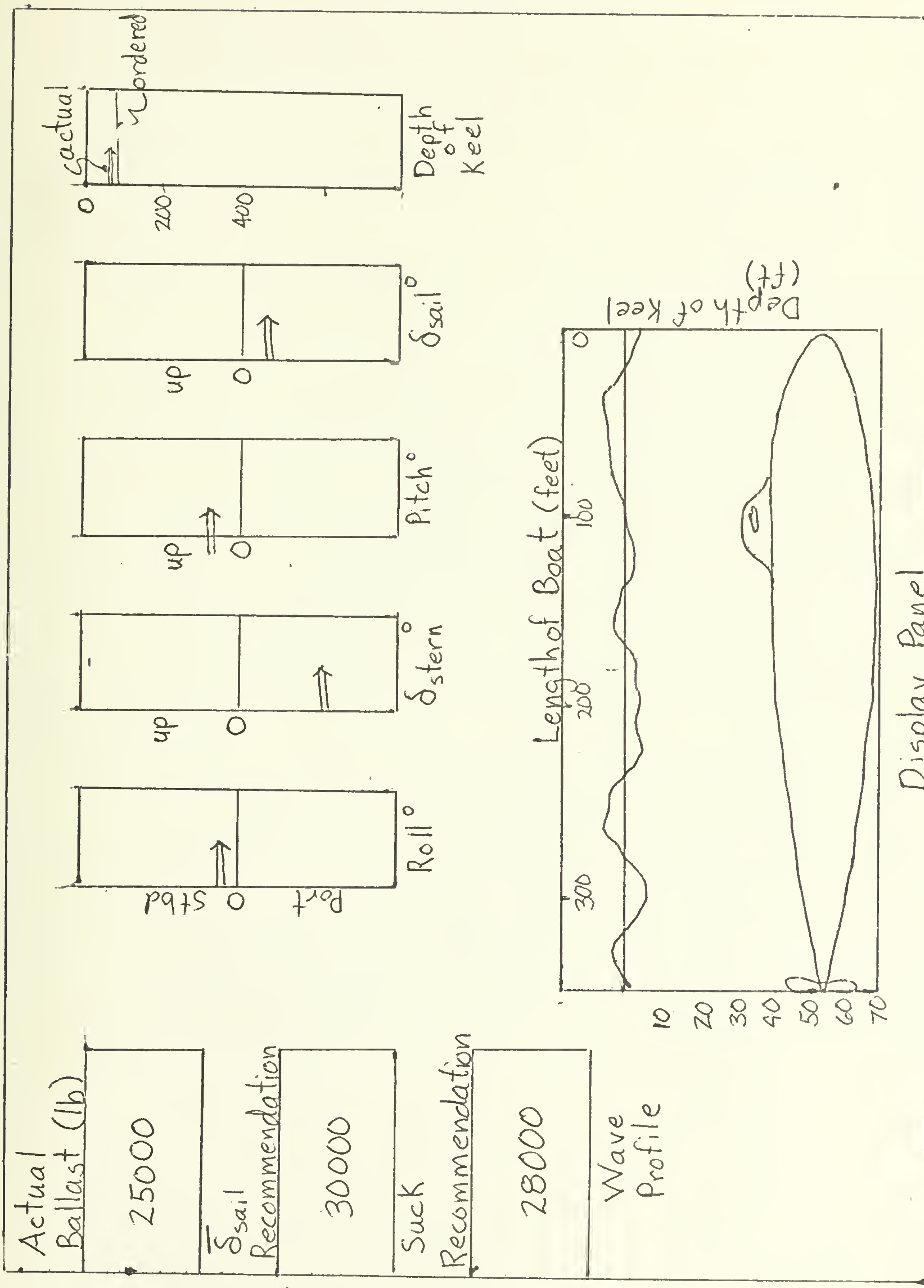
Figure 20

shaped waves. For instance, he would learn that when the main body is under a crest or a trough, motion will be primarily in pitch, and he should have his planes already moving in anticipation. (See figure 21 for control display panel.)

Another small computer begins a loop by monitoring the output of the sail plane and taking a time average of the amplitude of the deflections. This average is used to compute the amount of suck force the planes are being required to overcome. The weight discrepancy is displayed for the Diving Officer. Additionally, another computer can calculate the average suck by using the method of equation 31. This is converted to another weight recommendation. Either or both of these may be used. The Diving Officer is then free to compare these two recommendations with the actual ballast and make his own decition.

As in all of the previous methods, the two plane deflections, the sea excitation, and the ballast condition combine to determine the vehicle's response. The difference between input and output are again sensed and displayed while z and $\dot{z}$ are fed to the sail plane computer.

This system can be seen to be somewhat of a compromise between the other three. It uses the man from system one to control pitch, but it gives him an anticipatory capability similar to system three without the use of sonar. In the long run, the man may develop a better sense of accuracy than would the automatic system which had to rely upon so many approximations. (A computer can not give results which are more accurate than



Actual Ballast (lb)
25000
δ_{sail} Recommendation
30000
Suck Recommendation
28000
Wave Profile
Port
0
stbd
Roll°
up
0
δ_{stern}°
up
0
Pitch°
up
0
δ_{sail}°
0
200
400
actual
ordered
Depth of Keel
Length of Boat (feet)
100
200
300
Depth of Keel (ft)
0
10
20
30
40
50
60
70
Display Panel

Figure 21

the programs which make up its memory bank.)

Since the sail planes are in the free stream, it is possible to make a relatively accurate prediction of the angle of attack and thus the lift they are capable of procuding. There will still be a time lag in heave but it will be partially offset by the fact that a better estimation can be made of the effects of plane deflections. The system will take advantage of particle orbital velocities instead of blindly fighting them. This approach should allow the system to operate without the use of filters without exceeding existing hardware limits.

The calculations involved in this system are much simpler than those for system three and so the modeling and simulation costs will be substantially lower. Also the accuracy of the forecasted wave is much more reliable even though the method is much simpler.

Since the stern planesman must be constantly aware of his changing information, it would be impractical for him to operate the helm too. At least this will leave the helmsman free to concentrate on his job. As a back up for the sail plane computer, a man can use the same information as the stern planesman.

Conclusions and Recommendations

The manual system as it now exists certainly can continue as a suitable back up system. However, if operational speeds are to be increased and accuracy improved, a better solution must be found.

It seems apparent that a purely anticipatory automatic system such as the one used by hydrofoils is not practical. Even if the wave could be accurately predicted without the use of sonar, the highly complex configuration of a submarine as compared to that of the hydrofoil's three struts and three foils makes the analytic task for the computer nearly impossible with current analytical tools.

This leaves systems two and four. If the man in the loop is replaced by a machine, the reduced lag time can certainly be beneficial. However, even assuming that the necessary filters can be installed to limit excessive noise, the system will never be able to overcome the lag due to plane deflection rates.

System four appears to offer the advantages of each of the previous three systems plus some of its own.

1. No sonar.
2. Prediction of seaway before it reaches the control surfaces.
3. More information for the planesman.
4. Plenty of ballast information for the Diving Officer.
5. Manual control of stern plane but with anticipatory capability.

6. Computerized sail plane with no need of filters.
7. Comparitively reliable.
8. Comparitively low research and development costs
9. Releases three men.
10. Potentially quicker response and better accuracy than other three systems.

Tests should be conducted to insure that a man can be taught to anticipate the boat's response to various waves if they are visually displayed. If this is very successful, it might be worth the effort to try replacing the computer at the sail plane controls with a man. The problem is that he would not have nearly as much time to evaluate each approaching wave.

Appendix 1: Wave Sensing Sonar

In the process of trying to determine the shape of waves before thay reach the boat, a rather peculiar problem arises. Objects are normally sensed by sonar when an incident sonar wave is reflected back to the boat to be received. In this particular case however, geometry prohibits the reflected wave from returning to the boat. It is instead reflected away from the boat by the ocean's surface. However, due to the surface proximity, it would be possible to pick up backscattering created when the sonar pulse encounters the rough ocean surface.

Acoustic wave theory states that sufficient scattering will occur if the length of the incident wave is of the same order as the microroughness of the surface (length of the ocean's capillary waves).[14] A microscopic survey of the energy density spectrum reveals that these capillary waves are on the order of 0.01 feet.[4] Since the speed of sound in water is about 5000 fps, the length of the sonar pulse necessary is calculated as follows:

$$\lambda_{wave} \cong 0.01 = \lambda_{sonar} = c_{sound}/f_{sonar}$$

$$f_{sonar} = c_{sound}/0.01 = 500\text{ K Hz} \qquad (46)$$

Fortunately 500K Hz is a relatively common sonar frequency and would certainly be feasible. Therefore, for a submarine with a depth of keel of 60 feet, speed of 10 knots, and a forward search sonar inclined upwards at a 45 degree angle, the following situation exists. (See figure 22.)

A doppler sonar can be used to detect the oncoming wave

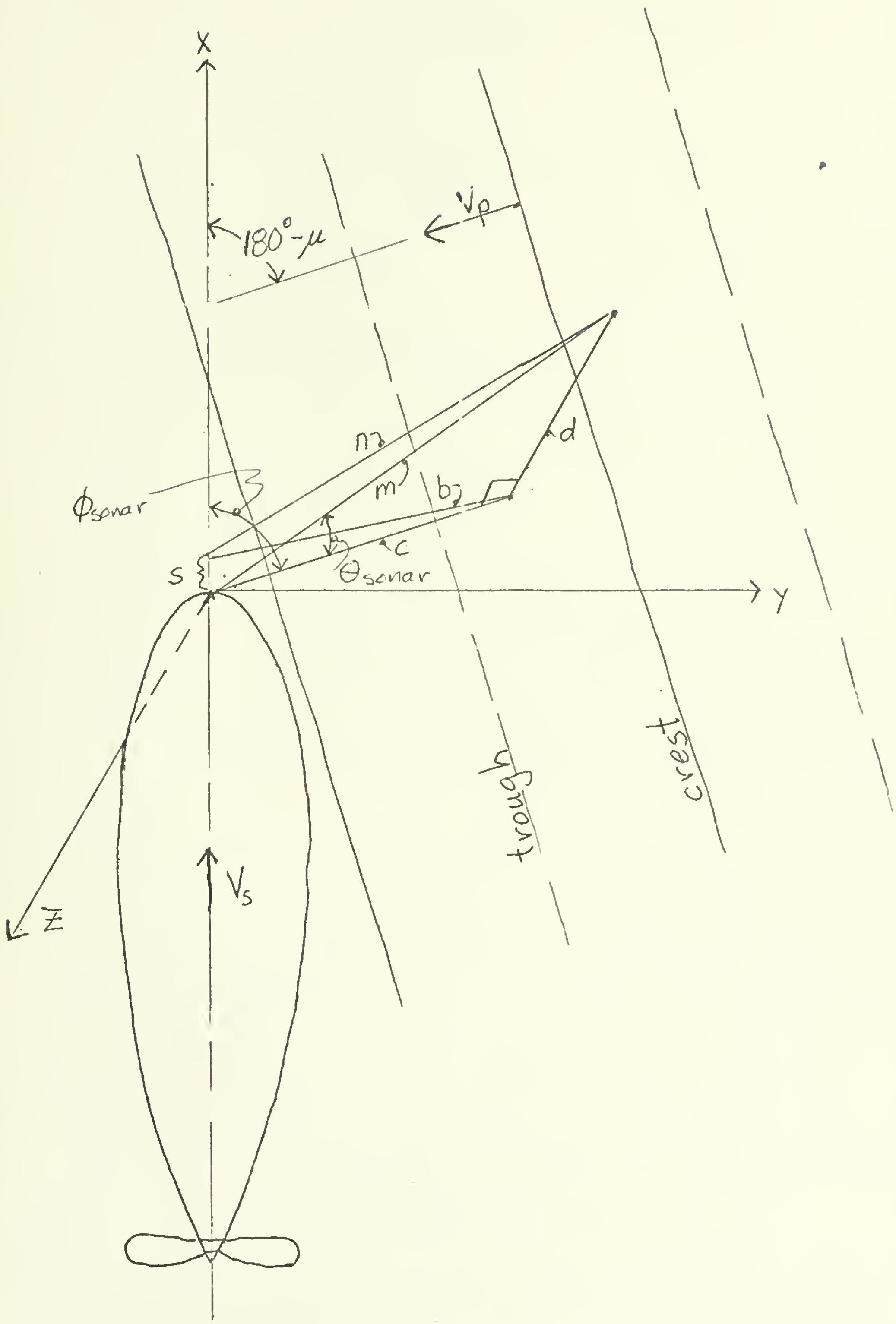


Figure 22

speed and thus frequency and length.

$$\Delta f = \text{frequency shift} = 2V_{relative} \times f / C_{sound} \qquad (47)$$

$$\Delta f = 2(V_s + V_p) f / C_{sound}$$

$$V_p = C_{sound} \times \Delta f / (2f) - V_s \qquad (48)$$

$$\omega = \sqrt{2\pi g / \lambda} \quad \text{and} \quad \lambda = V_p^2 \, 2\pi / g$$

$$\omega_e = (2\pi/\lambda)[V_p - V_s \cos\mu] \qquad (49)$$

For a doppler shift of 14.5K Hz,

$$V_p = 5000 \times 14.5 \times 10^3 / 2 \times 5 \times 10^5 - 16.7$$

$$V_p \cong 56 \text{ fps}$$

$$\lambda = 56^2 \times 2\pi / 32.2 = 628 \text{ ft}$$

$$\omega = \sqrt{2\pi g / 628} = 0.566 \text{ sec}^{-1}$$

$$\omega_e = 0.01(16.7 + 56) = 0.727 \text{ sec}^{-1}$$

Time to encounter is T=45ft/56fps = 0.8 seconds.

To increase this time to encounter, the angle merely needs to lessen. For the same wave, if a 3 second margin is desired,

$$T = b / V_p \qquad (50)$$

$$b = T / V_p$$

$$b = 168 \text{ ft}$$

$$\theta_{sonar} = \arctan \frac{d}{b}$$

$$\theta_{sonar} \cong 15^\circ$$

Since the path length of a pulse is short compared to the speed of sound in water, it can be assumed that the wave has not moved when the signal is received.

The problem now becomes one of determining the wave amplitude. If the boat is not moving perpendicular to the crests of the waves, the angle of attack into the waves becomes

important. This angle, μ, can be found by having the doppler sonar continuously scan the horizon to maximize the doppler shift. The angle ϕ_{sonar} at which this shift is maximized will also be the angle of encounter, μ. (See figure 22.)

Since the boat travels such a short distance between transmission and reception, s can be ignored thereby making m=n and b=c.

$$m = \Delta t \times c_{sound}/2 \tag{51}$$

where Δt is the elapsed time between transmission and reception.

ϕ is the angle where Δf is maximum and θ is chosen to allow for the correct lead time as shown earlier. Therefore

$$d = m \sin\theta \tag{52}$$

The wave amplitude then is $d - \overline{z}_{sub}$

If the boat were to choose a course into or with the seas, the entire boat would pass through the same part of each wave. These courses also would legitimately uncouple roll from the equations of motion by eliminating it. Unfortunately this is not always a feasible idea for a combatant submarine. Therefore, more problems occur for waves when $\mu \neq 0$ or 180 degrees. For such waves (assumed regular)

$$d = d_{max} \cos(\omega_e t - kx + \epsilon) \tag{53}$$

The included kx will apply to water particle orbital velocities at points aft of the bow. These are the velocities to be used for the linear strip theory.

For angles of ϕ between 20 and 160 degrees and between

200 and 340 degrees, it would probably be necessary to locate at least one other transmitter and receiver farther aft since the entire boat will not pass under the same wave shape in an irregular sea.

Appendix 2: Equations of Motion

Any free body has six degrees of freedom, three translational and three rotational. Any motion can be described as a combination of these six motions. Each degree of freedom can be affected by all of the other five. The problem that this creates can be seen from the fact that it becomes necessary to solve a simultaneous system of six partial differential equations. This obviously is not an enviable task.

Fortunately these may be broken into two catagories without serious damage. For motions in the horizontal plane, yaw, sway, surge, and roll only need be considered. For those motions in the vertical plane, roll, pitch, heave, and surge are applicable. The method reduces the problem to 4 equations. For a completely submerged submarine, the effects of surge are small enough to be neglected.

The problem of roll for a submarine near the surface is worthy of an entire paper by itself. For this reason, it will be neglected to simplify matters. For a submarine in head or following seas, this is a very valid step. Since due to the port and starboard symmetry, neither heave nor pitch will excite roll or vice versa. Problems do occur in seas which are not perpendicular to the path of travel. For these seas, very serious coupling can occur between roll and pitch and heave.

From reference 1 the equations of motion for the vertical plane are:

Surge $$X = m[\dot{u} + qw - rv - x_g(q^2+r^2) + y_g(pq-\dot{r}) + z_g(pr+\dot{q})] \qquad (54a)$$

Heave $Z = m[\dot{w} + pv - qu - z_g(p^2+q^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})]$ (54b)

Pitch $M = I_y\dot{q} + (I_x - I_z)rp + m[z_g(\dot{u} + qw - rv) - x_g(\dot{w} + pv - qu)$
$+ y_g z_g(qp - \dot{r}) - y_g x_g(qr - \dot{p}) + x_g z_g(p^2 - r^2)]$ (54c)

Roll has been decoupled. Assuming that motion is restricted to only pitch and heave and noting that $y_g = 0$,

$$Z = m[\dot{w} - z_g q^2 - x_g \dot{q}]$$
$$M = I_y \dot{q} + m[z_g q w - x_g \dot{w}] \qquad (55)$$

The equations are linearized in the following way.

$$q_0 = \dot{q}_0 = w_0 = \dot{w}_0 = 0 \qquad (56)$$

Since $\dot{q} = \dot{q}_0 + \Delta\dot{q}$; $q = \Delta q$, $w = \Delta w$, $\dot{q} = \Delta\dot{q}$, $\dot{w} = \Delta\dot{w}$

All terms containing products such as $\Delta q \Delta w$ will be dropped since they are second order and are therefore considered negligibly small.

$$Z = m\dot{w} - m x_g \dot{q}$$
$$M = I_y \dot{q} - m x_g \dot{w} \qquad (57)$$

For straight ahead motion at constant speed, these two equations must also contain the following dynamic response terms.

$$Z_\theta\theta + Z_q q + Z_{\dot{q}}\dot{q} + Z_z z + Z_w w + Z_{\dot{w}}\dot{w} \;\&\; M_\theta\theta + M_q q + M_{\dot{q}}\dot{q} + M_z z + M_w w + M_{\dot{w}}\dot{w} \qquad (58)$$

Z_θ is the partial derivative of the Z force with respect to θ evaluated at θ equal 0. Some of the partial derivatives are intuitively zero for a submarine at great depths, Z_z for instance. Near the surface however, the Z force varies with depth even in calm seas. (See figure 2?) The mere presence of $Z_z z$ alone is sufficient to make the system unstable in the vertical plane. Combining equations 57 and 58 gives:

Z: $-Z_z z - Z_w w + (m - Z_{\dot{w}})\dot{w} - Z_q q - Z_\theta\theta - (m x_g + Z_{\dot{q}})\dot{q} = 0$ (59a)

M: $-M_z z - M_w w + (m x_g - M_{\dot{w}})\dot{w} - M_\theta \theta - M_q q + (I_y - M_{\dot{q}})\dot{q} = 0$ (59b)

Further complications arise when the boat is placed beneath a seaway as indicated in an earlier section. As was noted at the time, the frequencies of heave and pitch will be identical to those of the encountered excitation. So

$$\left.\begin{aligned} Z_{excit} &= Z_{max} \sin(\omega_e t + \epsilon) \\ M_{excit} &= M_{max} \cos(\omega_e t + \epsilon) \end{aligned}\right\} \qquad (60)$$

for a regular long crested wave. Z_{max} can be found experimentally in the towing tank. For a submarine under a long crested wave at constant speed with fixed control surfaces

$$\left.\begin{aligned} -Z_z z - Z_w w + (m - Z_{\dot{w}})\dot{w} - Z_\theta \theta - Z_q q - (m x_g + Z_{\dot{q}})\dot{q} &= Z_{max} \sin(\omega_e t + \epsilon) \\ -M_z z - M_w w + (m x_g - M_{\dot{w}})\dot{w} - M_\theta \theta - M_q q + (I_y - M_{\dot{q}})\dot{q} &= M_{max} \cos(\omega_e t + \epsilon) \end{aligned}\right\} \qquad (61)$$

More complications arise when a stochastic excitation is introduced. Assuming linear superposition,

$$\begin{aligned} -Z_z z - Z_w w + (m - Z_{\dot{w}})\dot{w} - Z_\theta \theta - Z_q q - (m x_g + Z_{\dot{q}})\dot{q} &= \sum_{i=1} Z_i \sin(\omega_{e_i} t + \epsilon_i) \\ -M_z z - M_w w + (m x_g - M_{\dot{w}})\dot{w} - M_\theta \theta - M_q q + (I_y - M_{\dot{q}})\dot{q} &= \sum_{i=1} M_i \cos(\omega_{e_i} t + \epsilon_i) \end{aligned} \qquad (62)$$

Since model tests reveal, $(Z/h)_i$ = constant$_i$ and $(M/h)_i$ = constant$_i$

$$\begin{aligned} -Z_z z - Z_w w + (m - Z_{\dot{w}})\dot{w} - Z_\theta \theta - Z_q q - (m x_g + Z_{\dot{q}})\dot{q} &= \sum_{i=1} C_{1i} h_i \sin(\omega_{e_i} t + \epsilon_i) \\ -M_z z - M_w w + (m x_g - M_{\dot{w}})\dot{w} - M_\theta \theta - M_q q + (I_y - M_{\dot{q}})\dot{q} &= \sum_{i=1} C_{2i} h_i \cos(\omega_{e_i} t + \epsilon_i) \end{aligned} \qquad (63)$$

This infinite sum is impossible to work with so it will be replaced by some known frequency and wave height. In the previous appendix it was seen that the sonar was constantly detecting different waves. Therefore these will be used in determining a value for h and ω_e. The constant, C_i, is different for every ω_{e_i}. But knowing ω_e, this constant can be picked from the standard Response Amplitude Curves of $(Z/h)^2$ vs. ω_e.

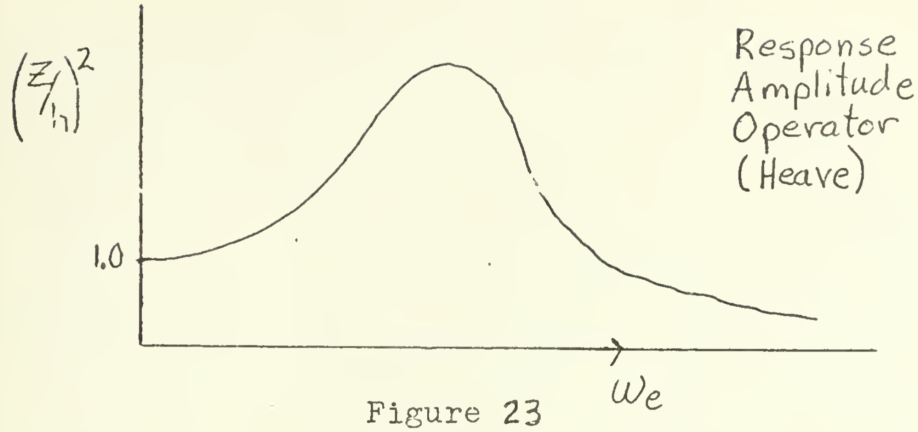


Figure 23

New constants, wave heights, and encountering frequencies will be computed periodically.

It is evident that deflection of the control surfaces is now necessary to counter the sinusoidal excitation. The following additional terms must now be added.

$$\begin{aligned} &Z_\delta \delta_{sail} + Z_{\dot\delta}\dot\delta_{sail} + Z_{\ddot\delta}\ddot\delta_{sail} + Z_\delta \delta_{stern} + Z_{\dot\delta}\dot\delta_{stern} + Z_{\ddot\delta}\ddot\delta_{stern} \\ &M_\delta \delta_{sail} + M_{\dot\delta}\dot\delta_{sail} + M_{\ddot\delta}\ddot\delta_{sail} + M_\delta \delta_{stern} + M_{\dot\delta}\dot\delta_{stern} + M_{\ddot\delta}\ddot\delta_{stern} \end{aligned} \tag{64}$$

The effect of plane velocity and acceleration is usually neglected leaving only

$$\begin{aligned} &-Z_z z - Z_w w + (m - Z_{\dot w})\dot w - Z_\theta \theta - Z_q q - (m x_G + Z_{\dot q})\dot q - C_{1i} h_i \sin(\omega_{e_i} t + \epsilon_i) = Z_\delta \delta_{sail} + Z_\delta \delta_{stern} \\ &-M_z z - M_w w + (m x_G - M_{\dot w})\dot w - M_\theta \theta - M_q q + (I_y - M_{\dot q})\dot q - C_{2i} h_i \cos(\omega_{e_i} t + \epsilon_i) = M_\delta \delta_{sail} + M_\delta \delta_{stern} \end{aligned} \tag{65}$$

In the above equations, the excitation has not yet arrived at the boat (using the anticipatory system). The thing to do now then is to order the sail and stern planes deflected soon enough before it arrives so that the planes have just reached the proper position as the excitation arrives. This requires solving the two equations for δ_{sail}, and δ_{stern}. This can be done using a Laplace Transform or some similar method.

Appendix 3: Strip Theory

In light of the problems encountered in applying the equations of motion, use of strip theory is probably a better tool for the near-surface case. This involves breaking the boat into a finite number of sections by passing planes through it perpendicular to the longitudinal axis. The dimensions of each section are assumed to be independent of the x coordinate. This requires that the length of each of the sections be small for the sake of accuracy.

Each of these sections is taken separately and the pressure is integrated over its surface to determine the forces and moments acting on this section. After this has been done for each section, the forces and moments of all the sections are summed to give the net force on the boat. Again only pitch and heave need be considered though all the other forces and moments may be calculated from F_{total} and M_{total}.

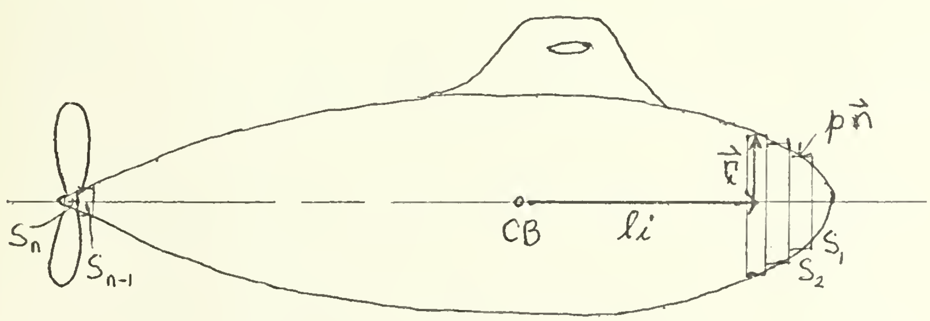


Figure 24

$$F_i = \iint_{S_i} p\,\vec{n}\,dS \tag{66}$$

$$F_{total} = \sum_{i=1}^{N} \iint_{S_i} p\vec{n}\,dS \tag{67}$$

Similarly, $$M_i = \iint_{S_i} p(\vec{r}\times\vec{n})\,dS_i \tag{68}$$

$$M_{total} = \sum_{i=1}^{N} \iint_{S_i} p(\vec{r}\times\vec{n})\,dS_i + \sum_{i=1}^{N} F_i l_i \tag{69}$$

where, as noted earlier, $$p = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}V_{tot}^2 + gz\right) \tag{70}$$

for $$V_{tot} = u_o + v_o + w_o \tag{71}$$

$$u_o = V_s + u_{max}\cos(\omega t)\cos\mu \tag{72a}$$

$$v_o = v_{max}\cos(\omega t)\sin\mu \tag{72b}$$

$$w_o = w_{max}\sin(\omega t) \tag{72c}$$

where $$u_{max} = v_{max} = w_{max} = A\omega\exp[-2\pi z_o/\lambda] \tag{73}$$

A and ω are those of the wave which was detected by the sonar.

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